2019

Montessori Short Bead Stairs, Developmental Delays, and Numeracy Skills of Preschoolers

Katrina R. Azevedo-Pinillos

MONTESSORI SHORT BEAD STAIRS, DEVELOPMENTAL DELAYS, AND NUMERACY SKILLS OF PRESCHOOLERS

DISSERTATION

Presented in Partial Fulfilment of the Requirements for

the Degree of Doctor of Philosophy in

Leadership and Education in

the Adrian Dominican School of Education of

Barry University

by

Katrina R. Azevedo-Pinillos, M.S.

*****

Barry University

2019

Area of Specialization: Exceptional Student Education

MONTESSORI SHORT BEAD STAIRS, DEVELOPMENTAL DELAYS, AND NUMERACY SKILLS OF PRESCHOOLERS

DISSERTATION

by

Katrina R. Azevedo-Pinillos

2019

APPROVED BY:

Judy Harris-Looby, PhD
Chairperson, Dissertation Committee

Gerene K. Starratt, PhD
Methodologist, Dissertation Committee

Javier Gonzalez, PhD
Member, Dissertation Committee

Jill Farrell, EdD
Dean, Adrian Dominican School of Education

Abstract

Montessori Short Bead Stairs, Developmental Delays, and Numeracy Skills of Preschoolers

Counting numbers is one of the first skills learned by children at an early age. Research indicates that children who master these skills in preschool, demonstrate a stronger conceptual understanding in mathematics later in life and are less likely to fall behind in mathematical skills (Gersten, Jordan, & Flojo, 2005; Purpura & Lonigan, 2015). More significantly, children with developmental delays often experience serious deficits in mathematics, which can be identified as early as 3-4 years old (Nguyen et al., 2016). Identifying deficits at an early age can identify children at risk of later academic difficulties or disabilities (Purpura, Reid, Eiland, & Baroody, 2015). Therefore, it is imperative to identify evidence-based instruction for teaching the acquisition of numbers and mastery of early numeracy skills for children ages 3-5 with and without developmental delays. This study investigated the impact of teaching two evidence-based instructional approaches, Montessori Short Bead Stairs and Traditional methods, on 159 preschool students with and without developmental delays. Participants' knowledge and performance of counting, identifying, ordering, and identifying the quantity of numbers 1-10 were examined using the Test of Early Mathematics Ability, 3rd Edition Form A and B for pre- and post-test comparisons. Additionally, 13 participating teachers completed a survey on their perceptions of their own confidence, reliability, willingness to use the lessons after the study, and perceptions of children's understanding of early numeracy skills. Results of this study suggest that randomization by classes yielded unequal treatment groups, calling into question treatment findings. However, children with developmental delays performed worse compared to their peers without developmental delays. Findings also shows similar improvement in both groups using both Montessori and Traditional curricula.

Table of Contents

Acknowledgements

This dissertation, although a representation of my dedication and hard work towards my career and passion in education, is truly a reflection and culmination of my educational experiences and efforts, the remarkable guidance of my Barry University professors and committee members, as well as the support of my family, close friends, and colleagues.

I would first like to express my gratitude to my dissertation committee. Thank you to my dissertation chair, Dr. Judy Harris-Looby, for your unwavering support through my masters and doctoral programs, and your guidance through this experience. To my methodologist, Dr. Gerene Starratt, for joining my committee without hesitation and guiding me with your professional insights, encouragement, and enthusiasm for reaching my goals throughout this dissertation process. And to Dr. Javier Gonzalez, whose passion for education and endless support throughout my bachelor's, master's, and doctoral program, served as a continual reminder of the importance of pursuing this terminal degree. My sincere appreciation to my committee for your commitment to my studies, expertise in the process, and your willingness to meet at a moment's notice in the final weeks that led to the dissertation defense.

I would also like to recognize a few professors who have positively impacted my journey at Barry University. Thank you, Dr. Cynthia Lasky, for being my "second mom" at Barry. You were instrumental in my decision to continue my graduate degrees at Barry and you never failed to remind me of my strengths and reaching my goals. Another special thank you to Dr. Priva Fischweicher, who not only challenged me educationally, but you were "one of my favorites" because your teaching forced me to work harder.

Thank you to all of my professors at Barry University and for your support along the way.

My family, close friends, and colleagues have endured a multitude of obstacles with me throughout this dissertation journey. I appreciate your support, love, and words of encouragement. To my parents, Estela and Carlos, I truly appreciate your patience and support in helping me with my daughters or simply being there for me so that I could keep going! You have been there from day one and have always been my biggest cheerleaders. To my daughters – Alexyss and Olivia – I am sorry for the time we have spent apart but thank you for reminding me that I am a great mother and I can do anything with you by my side. I hope to one day see both of you accomplishing your goals and never giving up. And to my better half, Nicolas, thank you for coming into my life and being the fire that kept me going throughout this journey. Your encouragement inspired and empowered me to be better. Finally, my sincerest gratefulness to every school, teacher, parent, and child who participated, as their direct willingness to participate will hopefully open the eyes of others to the importance of mathematics at an early age.

Dedication

Para minhas lindas filhas, com amor.

For my beautiful daughters, Alexyss Rosalie & Olivia Sofia.

I love you from my heart, to heaven, and back.

List of Tables

List of Figures

CHAPTER I

THE PROBLEM

Early numeracy skills, such as counting, one-to-one correspondence, sequencing, stable order, and cardinality are needed as a foundation for mastery in mathematics (Agrawal & Morin, 2016; Carpenter, Franke, Johnson, Turrou, & Wager, 2017; Hannula-Sormunen, Lehtinen, & Rasanen, 2015). More specifically, counting is a foundational concept required for further math skills that involves memorizing number words in a specific order, such as making quantitative comparisons (Hinton, Flores, Schweck, & Burton, 2016; Lee & Md-Yunus, 2016; Young-Loveridge, 2001). Furthermore, one-to-one correspondence is used to measure the set identity of a number so that students can connect one number with one object, and count with understanding (Izard, Streri, & Spelke, 2014). Once one-to-one correspondence is established, children can gain knowledge of cardinality. Counting that denotes quantity, or how many things are in a set (*i.e.,* one, two, or three), is known as cardinality or cardinal number knowledge (Dolschield, Winter, Ostrowski, & Penke, 2017). Therefore, cardinality is the last word in the counting sequence that names the quantity for that set. Although counting is the basis for understanding numerology, it is not exclusively related to cardinal number knowledge (Dolschield et al., 2017). Understanding the cardinality of a number is identified when a child improves their counting abilities (Shusterman, Slusser, & Odic, 2016). Once children learn to count, they have the ability to generalize the quantity of larger numbers (Cheung, Rubenson, & Barner, 2017).

Some children struggle with mathematical concepts due to a limited foundation of such concepts in the early developmental years (Aunio, Korhonen, Bashash, &

Khloshbakht, 2014; Mendizabal, Villagran, Guzman, & Hoyos, 2015). If children are provided with a strong foundation, the ability to close mathematical foundational gaps would be made possible (Agrawal & Morin, 2016; Fuson, Clements, & Sarama, 2015; Lee & Md-Yunus, 2016). With the presence of early mathematical skills, mathematics achievement can be predicted later in school (Hannula-Sormunen et al., 2015; Nguyen et al., 2016; Reardon & Portilla, 2016; Reid & Andrews, 2016; Wolfgang, Stannard, & Jones, 2001). Math achievement can be predicted based on early quantitative knowledge and cognitive skills (Chi, vanMarie, & Geary, 2016). Unfortunately, the research on preschool children and mathematics indicates that gender gaps still exist (Cimpian, Lubienski, Timmer, Makowski, & Miller, 2016; Tichenor, Welsh, Corcoran, Peichura, & Heins, 2016), motivation (Middleton & Spanias, 1999; Mutjaba & Reiss, 2016) and critical thinking skills (Haq & Alfilfili, 2015; Hu et al., 2016) are limited, and children are not prepared for kindergarten (Ansari & Winsler, 2016).

In mathematics, teachers use concrete materials, such as manipulatives, for teaching mathematical understanding and mathematics effectiveness (Carbonneau, Marley, & Selig, 2013; D'Angelo & Illiev, 2012; Furner & Worrell, 2017; Laski, Jor'dan, Daoust, & Murray, 2015; Rosli, Goldsby, & Capraro, 2015; Schneider et al., 2017). Manipulatives are used to improve number concepts for preschool children (Huntley-Fenner, 2001; Zhu, Chen, Li, & Deng, 2017). More specifically, preschool children have the ability to improve their counting performance (Peterson & McNeil, 2013), understanding of numbers (Huntley-Fenner, 2001), and early development of mathematical concepts (Fuson et al., 2015). Preschool children use manipulatives as symbols for learning (Chumark & Puncreobutr, 2016; Uttal, Scudder, & Deloache, 1997).

Children who struggle with mathematics can be identified at an early age (Purpura et al., 2015). Therefore, providing early intervention and teaching early numeracy skills using manipulatives for children, including children with deficits, are essential (Bashash, Outhred, & Bochner, 2003; Bennett & Rule, 2005; Bouck, Joshi, & Johnson, 2013; DuPaul, Kern, Caskie, Volpe, & Gilman, 2015; Gersten et al., 2005; Hudson, Zambone, & Brickhouse, 2016; King et al., 2016). Providing appropriate interventions and manipulatives are also important for students with a variety of developmental delays, such as children with math difficulties and learning disabilities (Bashash et al., 2003; Gersten et al., 2005; Gersten et al., 2009; Jimenez-Fernandez, 2015; Jordan & Levine, 2009; Lewis, 2016), attention deficit hyperactivity disorder (DuPaul et al., 2015), autism spectrum disorder (King et al., 2016), severe multiple disabilities (Hudson et al., 2016), and dyscalculia (Devine, Hill, Carey, & Szu'cs, 2018; Price, 2013). Research suggests that mathematical instruction with manipulatives can facilitate learning in all children (Fuson et al., 2015).

Constructivist theories are often used in mathematics instruction for children with learning disabilities (Xin, Liu, Jones, Tzur, & Sin, 2016). In particular, constructivist theorist Jean Piaget focused on a child's development process and how it relates to mathematics, specifically a child's need to learn numeracy skills (Ghazi & Ullah, 2015; Kose & Arslan, 2015; Lefa, 2014; Lefmann & Combs-Orme, 2013; Navarro, 2014; Ojose, 2008; Purpura & Lonigan, 2013). Educator and physician, Maria Montessori, also theorized about a child's development and its effect on preschool readiness (Kayili & Ari, 2011; Lillard, 2012; Lopata, Wallace, & Finn, 2005), and early math skills (Bauch & Hsu, 1988; Chisnall & Maher, 2007; Ongoren & Turcan, 2009) of typically developing children, as well as children with disabilities (Gitter, 1967; Pickering, 1992). By utilizing the theoretical frameworks of Piaget and Montessori, this study conducted a comparison between traditional and Montessori approaches to mathematical instruction of early numeracy skills on preschool children with and without developmental delays.

This introduction will explain the rationale for this study that includes an explanation of the development of early numeracy skills, the identification of early childhood policies and programming, a description of cognitive development and general knowledge, and the identification of children with developmental delays. The rationale also provided an explanation of evidence-based interventions in mathematics, the history and rationalization of the Montessori Methodology, and highlighted the theoretical foundations for this study. This section will conclude with a statement of the problem and purpose, specific research question and hypothesis, the significance of the study, and definitions for further understanding.

Early Numeracy Development

Counting is an early numeracy skill that is identified as one of the first basic skills a young child acquires (Carpenter et al., 2017; Kose & Arslan, 2015; Lefmann & Combs-Orme, 2013). Young children begin to learn to count through repetition and observation and eventually develop more complex mathematical skills and critical thinking. At age three and four, most children have the ability to count numbers and make connections with tangible items (Brueggemann & Gable, 2018; Carpenter et al., 2017; Purpura & Lonigan, 2015; Voustina, 2016).

According to the Florida Early Learning Coalition (2011), young children develop cognitive skills through stimulating environments via the process of discovery and exploration (Capriora & Anghelide, 2016; Kose & Arslan, 2015; Kyttala, Kanerva, & Kroesbergen, 2015; Lefmann & Combs-Orme, 2013; Roth, 2017; Steffe, 2016). Early numeracy skills include counting principles, one-to-one correspondence, stable-order, cardinality, sequencing of numbers, and basic number operations and mathematical understanding (Carpenter et al., 2017; Marmasse, Bletsas, & Marti, 2000; Reid, 2016). These components are essential for children to develop their understanding of early numeracy skills.

Preschool relations to the quantities of objects and cardinal knowledge are good predictors of mathematical achievement at the end of kindergarten (Chi et al., 2016). Unfortunately, some three and four-year-old children with delayed cognitive development lack the necessary mathematical processes needed for growth in mathematics (Aunio et al., 2014; Jimenez-Fernandez, 2015; Peterson & McNeil, 2013; Price, 2013; Purpura & Lonigan, 2013; Soydan, 2015; Xin et al., 2016). Delays in early numeracy skills and conceptual development continue to occur as these children mature (Agrawal & Morin, 2016; Hannula-Sormunen et al., 2015; Lee & Md-Yunus, 2016). According to the National Research Council, children entering kindergarten must have the prerequisite knowledge and skills related to number quantity, and educators are responsible for closing any learning gaps in the mastery of these mathematical skills (Fuson et al., 2015).

Early Childhood Education

Specific skills are needed for kindergarten readiness, such as counting to 20 and knowing the letters of the alphabet and letter sounds; however, some children lack these necessary skills. With a 50% decline in academic learning for children entering kindergarten (Reardon & Portilla, 2016), there is a need for children to be prepared for kindergarten. In a 2010 study by Reardon and Portilla (2016), more parents believed their children needed to be better prepared for kindergarten when compared to the parents of kindergarten children from previous years. This highlights the importance of emphasizing early learning gains prior to entering kindergarten, and the need to focus on early childhood learning initiatives.

Investments in early childhood education have grown tremendously in the past three decades. Since the 1990's, state spending on preschool initiatives has more than doubled to $5.6 billion in 2014 (Barnett, Carolan, Squires, Brown, & Horowitz, 2015; U.S. Census Bureau, 2015). With an increase in early childhood learning standards, early intervention, stricter state quality regulations for childcare providers, and improvements in accountability systems, the evolution of early childcare programs have been established.

Unfortunately, there is little empirical data to support whether stronger literacy and mathematical skills are evidenced in children entering kindergarten compared to years past. However, the evidence provided supports that children are "more ready" than before; especially those children attending pre-kindergarten at a public school compared to children who attended pre-kindergarten at a center-based childcare center (Ansari & Winsler, 2016; Bassok & Latham, 2016). Bassok and Latham (2016) utilized two national representative samples of children entering kindergarten in 1998 and 2010 to identify differences in educational performance among children ages three through six. The results of the study indicated that not only was there an increase in the percentage of parents who said their children recognized letters, but a 16% increase of parents stated that their children could count to 20 (Bassok & Latham, 2016). Although parental reports were solely utilized, the findings suggest that academic learning skills are acquired at an earlier age than they once were.

Conflicting results from Reardon and Portilla (2016) identify the need to understand if children are prepared for kindergarten. With higher expectations for the acquisition of more academic skills of children entering kindergarten, it is important to stress early childhood educational programming, provide responsive educational experiences, and narrow the "school readiness" gap (Ansari & Winsler, 2016).

Early Childhood Policies

Federal and state initiatives have driven public policy on early childhood education in the past century. As early as 1926, the National Association for the Education of Young Children (NAEYC) was established to improve the education and developmental services offered for children ages' birth to eight years old (NAEYC, 2010). The NAEYC supports policy development for young children and has promoted the adoption of programs and policies such as: Head Start, National Education Goals, the reauthorization of IDEA 2004, and the Early Learning Challenge (NAEYC, 2010).

Head start. By 1965, Head Start was developed as part of the United States Department of Health and Human Services to fund low-income children's social and cognitive development prior to kindergarten enrollment. In 2007, the *Improving Head Start for School Readiness Act* was reauthorized to strengthen the quality of school readiness goals, learning standards, teacher qualifications, and increased program monitoring across every state (Office of Head Start, 2017). As of today, Head Start has serviced over 32 million children and has evolved to be a respected preschool program in many communities across the United States (Office of Head Start, 2017). In a number of states, there have been many options for young children, such as half or full day programs, private or public, in-home care, and preschool or daycare facilities. Additionally, some states have universal preschool programs that are implemented in the public-school system. Universal preschool programs are government-funded preschool and available for all preschool-aged children.

National education goals. By the mid to late 1980's, a number of educational policies drove the need for high-quality programs. Goal 1 of the National Education Goals and responsiveness to learning reinforced the importance of early childhood education and experiences of children (National Education Goals Panel, 1999; West, 2017). A critical initiative by political leaders signaled the implementation of the "Ready to Learn" goal. This goal focused on the early childhood experiences and educational programs provided and their effect on educational success (National Education Goals Panel, 1999).

Reauthorization of IDEA 2004. Federal regulations emphasize the importance of special education programs. In 2004, the reauthorization of the *Individuals with Disabilities Education Act (IDEA)* highlighted Part B of the Act, stating that federal funds would be distributed for the funding of special education programs and related services for children with disabilities ages three through five. The 2004 reauthorization reinforced the importance of funding programs to address the young child. Part C of the Act denoted that states have a responsibility to service children ages birth to age two when necessary (U.S. Department of Education, 2018). As a result, the Office of Special Education and Rehabilitative Services (2016) provided $436 million to administer Part B and C programming for infants, toddlers, and young children with developmental disabilities or who have been diagnosed with a physical or mental condition. Based on these results, educational resources and services were provided to children who meet the criteria for services.

Early Learning Challenge. In 2009, the Early Learning Challenge fund was proposed to provide grants for local and state early childhood programs that serviced children birth through age five (Florida Early Learning Coalition, 2011). Grants encouraged states to raise their standards, promote program effectiveness and quality, and monitor early childhood program performance. However, the challenge developed approaches that would drive standards reform, implement and improve existing early learning programs, and ensure that children entering kindergarten are prepared academically, socially, emotionally, and behaviorally. The funding also promoted systems that facilitated screening and referrals for health, disability, and family support, as well as age- and developmentally appropriate curriculum practices (U.S. Department of Education, 2009).

Early Childhood Programming and Enrollment

Participation in preschool programs and enrollment in kindergarten nearly doubled from the early 1970's to the mid-1980's (Jamieson, Curry, & Martinez, 2001). These trends in early childhood education expected children to learn more prior to kindergarten and 1st grade. Expectations for academic achievement in the earlier school years increased, and increased retention in kindergarten was a way of adapting to the enhanced expectations (Shephard & Smith, 1988). However, limited data was collected on the kindergarten and early childhood educational programs, curricula, and teacher expectations during this time (West, 2017).

From 1998-2010, a significant drop from formal early childhood education to at-home parental care was documented for children from low-income homes (Bassok, Finch, Lee, Reardon, & Waldfogel, 2016). Yet, across all socioeconomic backgrounds, a shift from private early childcare centers to public center-based care was identified. Therefore, public center-based care was most commonly utilized for early childcare services.

In 2016, there were approximately 8,087,000 children ages three through five in the United States (Corcoran & Steinley, 2017). This total represented a vast majority of preschool-aged children enrolled in early childhood programs and facilities. Early childhood education programs were provided for educational experiences for children in preschool and kindergarten (National Center for Education Statistics, 2018). It is important to note that, according to educational statistics, at-home care and daycare facilities are not considered early childhood education and not included in the statistics for early childhood education.

As of 2015, the number of children enrolled in preschool education programs were greater for four (67%) and five-year old's (87%), than three-year old's (38%) (National Center for Education Statistics, 2018). Nearly half of these children attended full-day programs in preschool, and the number of children enrolled in kindergarten increased to 81%. Based on the findings, most preschool aged children attended an early childhood education program between the ages of four and five and increased enrollment upon entering kindergarten (National Center for Education Statistics, 2018). These children had the opportunity to be identified for early intervention services if needed. The vast majority that are not enrolled in early childhood education programs are reducing their chances of identification and servicing (Corcoran & Steinley, 2017).

Cognitive Development and General Knowledge

Three to five-year old children learn at various times throughout the day in daily routines, activities, play, socialization, and interactions with peers and adults. When provided stimulating environments and new experiences, children at this age are provided opportunities to develop new cognitive skills from those around them (Florida Early Learning Coalition, 2011). Children are given opportunities to take risks, make mistakes, discover possibilities, and explore. Based on a child's interest, learning takes place. A teacher has the role of introducing new concepts and integrating content for building a strong foundation. According to the Florida Early Learning Coalition (2011), the cognitive development and general knowledge domain for four-year-old children consists of life skills and processing to support learning, such as mathematical thinking, scientific inquiry, social studies, and creative expression through the arts. These domains are interrelated and support a variety of components needed for learning. As children learn, they make connections through classification, patterns, comparison and contrasts (Florida Early Learning Coalition, 2011). At a young age, children begin to develop reasoning skills, observations, predictions, and problem solving, all of which are needed for mathematical skills to develop. Cognitive development is not limited to schooling; children absorb from the world around them.

Children with Developmental Delays and Disabilities

Academic achievement gaps in mathematics exist between children with developmental disabilities and typically developing children (Agrawal & Morin, 2016). Children with developmental disabilities are more prone to have cognitive and skill deficits than typically developing children. Many of these children experience difficulties with conceptual and procedural knowledge of various mathematical concepts (Agrawal & Morin, 2016). With over one million children in Florida under the age of four in 2014, and 105,089 children in Broward County alone, the need to service children at a young age, particularly those with disabilities, is critical (Kids Count Data Center, 2015). As of 2018 and as it related to this study, there are close to 5,447 children under the age of five living in a community in Southeast, Florida where this study was conducted (U.S. Census Bureau, 2018). Of this population, 1,551 are children ages three to five.

Research in early mathematics growth identifies that by the age of five, children should know and demonstrate knowledge of early numeracy skills and concepts (West Virginia Department of Education, 2016). One-to-one correspondence, knowledge of counting, stable order, and cardinal numbers are early numeracy skills learned before a child enters kindergarten (Jordan & Levine, 2009; Purpura & Lonigan, 2015). Studies show that many children entering into kindergarten still lack the necessary pre-computational skills. According to the Florida Department of Education (2014), the following standards reflect the goals in mathematics for children in kindergarten:

> Know the names and count the sequence of numbers; count forward beginning from a given number with the known sequence (MAFS.K.CC.1.2); read numbers and represent a number of objects with a written numerical 0-20 (MAFS.K.CC.1.3); count to tell the number of objects (MAFS.K.CC.2.4); when counting objects, say

the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object; understand the relationship between the numbers and quantities; connect counting to cardinality (MAFS.K.CC.2.4); and understand that each successive number name refers to quantities that is one larger (MAFS.K.CC.2.5) (Florida Department of Education, 2014, pp. 1-4).

With an emphasis on high stakes testing and accountability, the national and state focus has shifted towards early intervention. Research supports the notion that when skills are targeted at an early age, at-risk children could be prevented from falling further behind (Gersten et al., 2005; Purpura et al., 2015). Therefore, the National Association for the Education of Young Children (NAEYC) (2010) and the National Council of Teachers of Mathematics (NCTM) (2003) have called for research on mathematics curriculum targeting preschool-aged children. Specifically, for children with developmental delays, such as a delay in mathematics, there is a need to identify curriculum that appropriately targets early numeracy skills.

According to the National Center for Education Statistics (2018), 6.7 million children were identified as having a disability in the United States in 2014; however, approximately 735,000 children in the United States were receiving special education services in (Digest of Education Statistics, 2016). The number of children with disabilities is 13 percent of all public-school students, with the majority of race/ethnicities being: American Indian/Alaska Native (17%), Black (16%), White (14%) two or more races (13%), Hispanic (12%), and Pacific Islander (12%) (National Center for Education Statistics, 2018). Most of these children were eligible for special education services under a specific learning disability category (34%), speech and language impairments (20%), and other health impaired (14%).

The statistics do not classify the number of children under the age of five in each category because children under this age are often undefined due to inaccuracy and deficiencies in reporting and are classified as having a "developmental delay" (Digest of Education Statistics, 2016). States often use this terminology, developmental delay, as an option to report differences for young children who were unidentifiable. Developmental delays include, but are not limited to, underachievement, or intellectual, physical, or deficits. One particular type of delay that may be included under the broad classification of a developmental delay in young children during the primary years is developmental dyscalculia. According to IDEA, the term "developmental delay" is most commonly used to describe a child who presents a delay up to nine years of age.

Developmental Dyscalculia

Children with developmental dyscalculia, also known as a mathematical learning disability, typically lag behind their peers in overall mathematical performance (Devine et al., 2018). Szu'cs and Goswani (2013) suggest that this impairment is caused by one or more of the following problems: working memory deficits, inhibition, spatial skills deficits, or phonological awareness problems. According to the American Psychiatric Association's (2013) *Diagnostic and Statistical Manual of Mental Disorders (5th ed.: DSM-5),* dyscalculia is clinically diagnosed as a "specific disorder of arithmetical skills". Children who are identified with developmental dyscalculia according to the DSM-5 definition must demonstrate mathematical abilities (as measured on standardized assessments) significantly below their age-level peers but is not related to an intellectual disability or inadequate instruction (American Psychiatric Association, 2013).

Clinical diagnostic criteria for developmental dyscalculia is not consistent in research (Devine, Soltesz, Nobes, Goswani, & Szu'cs, 2013; Devine et al., 2018; Szu'cs & Goswani, 2013). Most researchers define developmental dyscalculia operationally where children with lower mathematics performance than their age-level peers do not demonstrate poor performance in other subject areas (e.g., reading, language arts, etc.). Others have used the discrepancy model (e.g., discrepancy between mathematical performance and performance on a controlled variable such as IQ), which is a 2-year achievement delay or resistance to intervention (such as Response to Intervention: RtI) (Devine et al., 2013). Devine et al. (2013) researched the effects of using different definitions of developmental dyscalculia on the prevalence of the disability and gender differences. Results indicated that when mathematical performance was one standard deviation below the mean and reading performance was at or more than standard deviation below the mean, 6% of their participants met the criteria for developmental dyscalculia.

Summary of Children with Delays and Disabilities

Reverting to foundational concepts is imperative to improving the deficits in mathematics; therefore, identifying the basic concepts in preschool-aged children is crucial. As counting is an important concept for understanding numbers, mathematical concepts, and early numeracy skills, early intervention is needed at this level (Bashash et al., 2003; Jordan & Levine, 2009). At a young age, children can be identified as having mathematical delays, which may be classified as dyscalculia. Therefore, children with mathematical delays and those without can benefit from hands-on manipulatives to strengthen mathematical foundations (Agrawal & Morin, 2016; Jordan & Levine, 2009; Peterson & McNeil, 2013).

Evidence-Based Interventions in Mathematics

As hands-on evidence-based interventions are required in mathematics, it is essential to identify hands-on evidence-based interventions in traditional and Montessori settings. Concrete representational-abstract (CRA) is one evidence-based intervention used in mathematical instruction. This section will explain the concrete-representational-abstract intervention strategy and its importance in mathematics.

Concrete-Representational-Abstract

Concrete-representational-abstract (CRA) is a framework of mathematical instruction that includes hands-on, concrete objects or tools needed in the classroom (Agrawal & Morin, 2016; Carbonneau et al., 2013; Peterson & McNeil, 2013). This evidence-based approach includes a three-stage instructional sequence (concrete, representation, and abstract) in which the teacher models the concept and material, transforms the concrete model into a drawing, and models the concept in numbers and symbols (Flores, 2010). The use of concrete-representational-abstract (CRA) manipulatives and interventions are recommended to improve operational skills, counting principles, and overall knowledge and understanding of number concepts for typically developing children and children with developmental delays (Agrawal & Morin, 2016; Bashash et al., 2003; D'Angelo & Iliev, 2012; Hewitt, 2001; Hudson et al., 2016; Peterson & McNeil, 2013; Post, 1981; Rosli et al., 2015).

Agrawal and Morin (2016) recognized the use of concrete manipulatives to support the instruction of students with intellectual disabilities and connect their foundational understanding of mathematical skills and concepts. Similarly, Soydan (2015) indicated that six-year-old children acquire a higher degree of operational skills when presented with hands-on educational materials. Therefore, the CRA approach has the potential to improve operational skills for children with and without disabilities through the use of hands-on manipulatives.

This instructional approach is aligned with the Montessori and Piaget theories of learning. Both Piaget (1942) and Montessori (1949) developed theories that incorporated the use of concrete objects and a child's cognitive development (as cited in Capriora & Anghelide, 2016; Montessori, 1936; Montessori 1967; Pickering, 1992; Roth, 2017). Young children have difficulty thinking abstractly; therefore, these theorists argued that concrete materials should be used for young children through an experiential process (Capriora & Anghelide, 2016; McNeil & Uttal, 2009; Navarro, 2014; Steffe, 2016). However, CRA focuses solely on the teacher's instructional skills versus the child's learning abilities. Therefore, it is important to identify the components of the Montessori Method that can be effective for a preschool child's learning and development of early numeracy skills.

Montessori Methodology

Another approach that is designed to focus on a child's mathematical development of early numeracy skills and concepts at an early age is the Montessori Method. This section will explain the background of the Montessori Method for further understanding of the inception of the approach, the child's interaction with the environment, the Montessori approach and its processes, and an analysis of how the Montessori Method is effective in the development of the child.

Background of the Montessori Method

In 1894 (during the last two years of medical school), Maria Montessori studied at a pediatric hospital and gained experiences that prepared her foundation and theoretical perspective of learning (Gutek, 2004). Soon after, she researched intellectual and developmental delays (formally known as mental retardation), cognitive impairments, other psychological disorders in children. Based on the frameworks of Jean-Marc Gaspard Itard (1774-1838) and Eduord Seguin (1812-1880), Montessori developed her educational method (Gutek, 2004).

Itard and Sequin's contributions. Jean Marc Gaspard Itard was a French physician and psychologist who worked with deaf and hearing-impaired children. His research and specialties focused on transferring clinical observations of patients to educator observations. One of his most publicized works consisted of the "wild boy of Averyon" in which a feral boy was found in the woods and lived with animals (Itard, 1802). The boy was found without language skills, and Itard decided to educate and train the twelve-year old in speaking and practical life skills (Gutek, 2004). However, the boy resisted much of Itard's efforts. Itard was determined to prove that human beings go through very specific and necessary stages of human growth. His experience with children with mental impairments concluded that children needed to experience certain activities during the appropriate stage of development. Intrigued with Itard's position on clinical observations, Maria Montessori later utilized clinical observations in her own experimental psychology research (Gutek, 2004).

Another prominent French physician who worked with children with mental impairments was Eduord Seguin. Sequin worked with children in insane asylums and believed that these centers should be used for training and education, and that medical and theoretical knowledge should be combined to treat such impairments (Gutek, 2004). Seguin formulated various didactic materials (instructional materials) to train a child's senses and improve physical skills. Montessori adopted various techniques from Sequin's experimental research, such as focusing instruction on the developmental stages and using didactic materials for teaching and training children to learn practice skills that could be achieved independently (Montessori, 1964). Based on the theoretical foundations and clinical observations of Itard and Sequin, Montessori developed two specific principles that served as the basis for her research: (1) children with mental impairments required special kinds of education, not solely medical treatments; and (2) this kind of education required the use of didactic materials (Gutek, 2004).

Casa dei bambini. In 1907, Montessori opened her first school, the *Casa dei Bambini* (also known as the Children's House), for 50 children ages three to seven living in one of Rome's poorest neighborhoods in the San Lorenzo District (Gutek, 2004; Montessori, 1964). By opening a school in an impoverished area, Montessori attempted to ameliorate the anguish of the poor through humanitarian and educational means. Montessori's method was founded in the *Casa dei Bambini* as she combined sociological and educational views (Gutek, 2004). The connection between education and family played a major role in the Montessori Methodology as the school served as a foundation for socializing a family and a house served to connect with the community. As a place for a young child's education, *Casa dei Bambini* contained a structured and well-ordered environment; this set the stage for the Montessori environment (Montessori, 1964).

Montessori Environment

A Montessori classroom is strategically arranged and organized with tables, chairs, and various equipment suited for a child's needs (Pickering, 1992). Different from a traditional classroom, the Montessori classroom was organized to allow a child to move freely around the classroom. Tables, chairs, washstands, and other types of classroom furniture were sized for child's use. Low cupboards housed easy-to-reach didactic materials for children to remove and return on shelves. Each tool and material were designed to cultivate sensory, fine motor, gross motor skills, and the independence and self-confidence in performing skills on their own (Gutek, 2004; Montessori, 1964).

Sensitive periods. The curriculum of Montessori is arranged according to a child's development called "sensitive periods". These sensitive periods refer to the child's readiness to experience and participate in learning activities (Montessori, 1964). To assist with the child's development during these periods, children are provided with didactic materials that allow the child to correct themselves. Self-motivation is cultivated when children have the ability to select their own materials and activities (Montessori, 1967). Because children use materials that are self-correcting, each child works at his/her own pace and does not require much of the teacher's attention, correction, or assistance.

The materials used in the Montessori classroom are designed to provide children with self-discipline and self-reliance as they have the ability to identify mistakes and repeat tasks for mastery (Gutek, 2004). Based on this theory, Montessori designed a curriculum that derived from the observations and experimentations of a child to develop practical life skills, sensory and motor training, and other skills in the academic subject areas (Montessori, 1964. Although these ideals were not in place at the opening of the *Casa dei Bambini*, Montessori completed her method of education with each of these components in mind (Gutek, 2004).

Montessori Approach

Montessori designed exercises that developed practical life skills, such as washing dishes, tying shoelaces, or buttoning a shirt. Children practiced these skills, and once mastered, could transfer their knowledge to everyday life. The purpose of developing these skills was to gain independence and self-confidence without depending upon an adult (Frierson, 2016).

Sensory training was also designed for children to make clear distinctions between colors, sounds, tones, and manipulation of a variety of objects for comparisons and contrasts (Montessori, 1967). Designed with a specific order in mind, Montessori developed materials that began with a series of materials (Montessori, 1964. For example, the first of the sets are inserting wooden cylinders of different sizes into the same size wooden blocks. Following these insets are the ten pink wooden cubes of various sizes which a child uses to build a tower, known as the Pink Tower. These series of sets continue with geometric solids, wooden plane sets, musical tone bells, and other didactic materials. The purpose of these sets is set to distinguish comparisons and contrasts between objects (Pickering, 1992).

Language development is also addressed in the Montessori Method. As teaching reading and writing pose a variety of difficulties, Montessori proposed that when children were ready to learn to read and write, they would develop the skills needed (Montessori, 1964). Her experiments helped her to create appropriate materials needed for reading, writing, and arithmetic. These materials include sandpaper letters, cardboard letters and numbers, counting rods, and strings of beads of different colors. Through language development, Montessori proposed that children learned to count by arranging objects in a specific order and measuring the number according to a series of colored rods of different lengths (Gutek, 2004).

Overview of the Montessori Method

Montessori is a child-centered approach in which tools and materials are specifically designed for young children and the teacher serves as a facilitator assisting children to reach their fullest potential (Chisnall & Maher, 2007). In a multi-age classroom, children observe and absorb new concepts from other activities of other children. This early childhood-centered approach promotes order, organization, and foundational skills to introduce, teach, and engage students in the mastery of skills.

Children in a Montessori classroom have access to formulated materials designed to instruct and support visual and hands-on investigation of concepts (Chisnall & Maher, 2007). The Montessori Method and mathematical materials focus on quantities, relationships, and patterns in an effort to develop abstract reasoning (Pickering, 1992). This methodology utilizes manipulatives in sequential order to introduce and to master basic counting skills in typically developing children and children with developmental delays.

Theoretical Foundations Relative to Learning Mathematics

Overall, this study focused on the foundations of mathematics and how it relates to constructivist perspectives in education. The overarching theory for this study was based on Jean Piaget's Stages of Cognitive Development. Piaget proposed that children undergo four stages of development: sensorimotor, preoperational, concrete operational, and formal operations. He highlighted the sensorimotor stage of children birth to 2 years of age and their unique ability to develop an understanding of the concepts of numbers and counting (Kose & Arslan, 2015). He also pointed out that it is important for children 2 to 7 years old to absorb language during the preoperational stage (Capriora & Angehlide, 2016; Lefmann & Combs-Orme, 2013; Navarro, 2014; Steffe, 2016).

Stemming from Montessori's overall explanation of a child's development, she based her theory on the observations of children, research from Itard and Sequin, and her assumptions about a child's growth, development, and education (as cited by Gutek, 2004). The theoretical foundations of Itard (1802), Sequin (1846), and Montessori (1949) serve as the foundation of the current study because it highlights the importance of a child's development in learning. According to Montessori, children experience four *Planes of Development* (infancy, childhood, adolescence, and transition to adulthood) similar to Piaget's stages of development. Montessori explained that the first plane (ages birth to six) is where children are most attentive to organization and the classification of objects (Montessori, 1964). The second plane (ages seven to twelve), children were most attentive to organization and the classification of abstractions. For the purpose of this study, the focus was on the first plane of development whereby children are most attentive to organization and the classification of objects – a foundation for early numeracy skills.

Montessori applied the scientific method to her study of child development with a specific focus on the patterns in which children develop (Frierson, 2016). This mindset allowed her to create an educational environment and a set of instructional processes that emphasized patterns in human growth and development. The educational process embraced the importance and connections among the role of the child, the teacher, the environment, and the approach. Children interacted with the environment and adapted to objects and situations within the environment (Gutek, 2004).

Both Montessori and Piaget proposed that when a child was ready to learn new skills, he/she could master those skills in their environment. Through the use of didactic materials, or manipulatives, children have the ability to master practical life skills and educational processes (Montessori, 1964). In order to master these skills, children are provided with self-confidence and independence as a child no longer depends upon the teacher. To achieve mastery, Montessori believes children have an absorbent mind in which at an early phase in the stages of development, children function unconsciously and learn by interacting with the environment (Gutek, 2004).

Overall, Montessori proposed that through the understanding of the stages of development and in a prepared environment, young children have an absorbent mind and learn from the stimulating environment around them (Montessori, 1967). This served as the theoretical framework of this study for preschool children, specifically children with developmental delays, and their ability to master numeration skills using didactic materials prepared in a series of instructional processes.

Theoretical perspectives from Itard, Sequin, Piaget, and Montessori collectively describe a child's foundation and appropriate stages for learning mathematics. Through the foundations of constructivist viewpoints and Montessori and Piaget's theories of cognitive development, this framework distinguished the role of the teacher (Brooks & Brooks, 1999; Caprioara & Angehilde, 2016; Roth, 2017; Steffe, 2016), the students' engagement with mathematics (Roth, 2017; Steffe, 2016), and how numbers are processed (Cakiroglu & Taskin, 2016; Navarro, 2014).

Statement of the Problem

Although mathematics is mostly an abstract discipline, the role of manipulatives at an early age is essential (Post, 1981). Research has been conducted on the use of manipulatives for over 30 years and continues to identify specific manipulatives that prove to be effective for learning. In 1981, Post identified the need to explore how learning and teaching styles, materials, relationships between materials and content, and the order of manipulative representation interact among themselves. These same concerns still exist today.

In order to reduce the number of children with lingering mathematical learning disabilities, or dyscalculia, it is important to identify deficits at an early age and provide early interventions. As counting is one of the foundational and basic components of mathematics, children must learn to count appropriately and achieve mastery in this skill (Bashash et al., 2003). However, concerns about early prediction and early identification of mathematical deficits have risen. Researchers have questioned the extent to which preschool mathematical competencies predict mathematical achievement in later years (Foreman & Gubbinns, 2015). The concern is whether we can identify children with disabilities in mathematics as early as three to five years of age based on basic counting skills and early numeracy development. Nguyen et al. (2016) identified early numeracy abilities (such as counting and number identification) to be the strongest predictors of later mathematical achievement. Research suggests that the early competencies of children in preschool set the course for later academic achievement in mathematics (Jordan & Levine, 2009; Purpura et al., 2015; Reid & Andrews, 2016; Wolfgang et al., 2001), suggesting that it is important to identify evidence-based practices that support and help develop preschool-aged children's knowledge and performance in counting.

One of the main components in introducing the decimal system in mathematics for the Montessori Method is the use of the Golden Beads in the Short Bead Stairs. Golden Beads are concrete tools that represent a number. Using the Golden Beads, numbers one through nine are represented with individual units. Although children are introduced to these materials and can count the beads, children often do not understand the one-to-one correspondence with the Golden Beads alone (Glermain, 2008). As a result, Montessori developed the Short Bead Stair materials to help children grasp the idea of quantity. Short Bead Stairs are a series of beads and wooden cards used for teaching counting, identifying numbers, identifying quantity, and identifying the order of numbers. These educational tools introduce place value in the decimal system to grasp the idea of quantity using a set of colored glass beads representing an appropriate number (Glermain, 2008; Montessori, 1964). Short Bead Stairs are initially used to teach children how to count numbers one through nine and are further used for introducing addition and subtraction (Milinkovic & Bogavac, 2013). A number of studies show that there is a link between early numeracy skills and later mathematics achievement (Chi et al., 2016; Nguyen et al., 2016). However, no one has specifically focused on the impact of Short Bead Stairs on a child's mathematical ability.

This research study utilized the theoretical components of Piaget's Theory of Cognitive Development and Montessori's Planes of Development to explain the development of mathematical components of reasoning. Focused on the child's environment, the role of the student and the teacher, the cognitive knowledge, and the use of concrete tools and manipulatives, Piaget and Montessori's theories of cognitive development set the foundations for this research study. By understanding the process in which children develop and the comparisons between traditional and Montessori environments, this theoretical framework established the basis for this study.

Because counting is a skill developed at an early age and is essential for the development of mathematics skills, identification of early mathematical interventions and evidence-based practices are needed. Research supports the use of manipulatives and concrete-abstract representational (CRA) interventions to improve operational skills, counting principles, and overall knowledge and understanding of number concepts for typically developing children and children with developmental delays (Agrawal & Morin, 2016; Bashash et al., 2003; D'Angelo & Iliev, 2012; Hewitt, 2001; Hudson et al., 2016; Peterson & McNeil, 2013; Post, 1981; Rosli et al., 2015); however, some researchers find inconsistent results to similar practices (Jimenez-Fernandez, 2015; King, Lemons, & Davidson, 2016; Laski et al., 2015; Soydan, 2015). One approach that was initially developed for children with disabilities but lacks sufficient empirical data is the use of the Montessori Method. Recent studies have supported the use of Montessori manipulatives and methodology because of its formulated color-coordinated tools and structured process of learning (Bennett & Rule, 2005); however, systematic evaluation of the effectiveness of these approaches has not yet been conducted. Montessori and Piaget's stages of development provided a foundation for the basis of this study. Their theories explained the connection between a child's environment and hands-on materials in a child's development.

This study specifically explored the impact of preschool children's use of the Montessori Short Bead Stairs on counting, identifying, ordering, and the quantity of numbers. Counting uses working memory to verbally identify numbers (Kroesbergen, van't Noordende, & Kolkman, 2014; Kyttala et al., 2015; Pinhas, Donoahue, Woldorff, & Brannon, 2014). This process is needed to name and count pictures (Wong, 2017); however, it is not exclusively related to cardinality (Dolschield et al., 2017). Cardinality is the process in which a child improves his number counting skills and places numbers in a specific order (Shusterman et al., 2016). The quantification of numbers, or understanding the amount of each number, takes place after counting, identifying, and ordering has been established (Dolschield et al., 2017; Lee & Md-Yunus, 2016).

Statement of the Purpose

The purpose of this study was to investigate the impact of using Montessori Short Bead Stairs (used for counting numbers 1-10) on preschool children's (with and without developmental delays) numeracy skills as measured by their ability to count, identify numbers (one-to-one correspondence), order numbers (cardinality), and quantify numbers. The aim of this study was to determine if the use of the Montessori Short Bead Stairs specifically for the teaching of numeracy skills was more effective than the traditional teaching methods for children with and without developmental delays.

Research Question and Hypotheses

Based on the purpose of this study, the following research question with corresponding research hypotheses guided this study:

Research Question: Is there a difference in the performance of children, with and without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in counting, identifying, ordering, and identifying the quantity of numbers 1-10?

H_a1: There will be a difference in the performance of preschool children, with and without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in *counting* numbers 1-10.

H_o1: There will be no difference in the performance of preschool children, with or without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in counting numbers 1-10.

H_a2: There will be a difference in the performance of preschool children, with and without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in *identifying* numbers 1-10.

H_o2: There will be no difference in the performance of preschool children, with or without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in identifying numbers 1-10.

H_a3: There will be a difference in the performance of preschool children, with and without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in *ordering* numbers 1-10.

H_o3: There will be no difference in the performance of preschool children, with or without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in ordering numbers 1-10.

H_a4: There will be a difference in the performance of preschool children, with and without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in *identifying the quantity* of numbers 1-10.

H_o4: There will be no difference in the performance of preschool children, with or without developmental delays, who used the Montessori Short Bead Stairs and those taught by traditional non-Montessori methods in identifying the quantity of numbers 1-10.

Definition of Terms

Cardinality / Cardinal Numbers: Cardinal numbers represent how much of something there is. A number that represents a quantity (one, two, three, etc.; Dolschield et al., 2017).

Constructivism: A view in mathematics that is based on observation and science of how people learn, understand, and construct knowledge (Roth, 2017; Steffe, 2016). Constructivists believe that understanding and knowledge is based on the experiences of things in the world.

Conservation of Numbers: The ability to recognize that different distributions of objects in one specific place has no effect on the quantity (Cakiroglu & Taskin, 2016).

Counting: A common numerical activity and a foundational skill, counting is reaching a total number (Hinton et al., 2016).

Developmental Delay: A condition of a child being less developed mentally or physically than is normal for its age (Boyle et al., 2011).

Developmental Disability: A group of chronic conditions that is due to a person's mental and/or physical impairments such as language, mobility, learning, self-help, and independent living. Developmental disabilities begin during any stage of development and last throughout a person's lifetime (Boyle et al., 2011).

Developmental Dyscalculia: A selective and serious deficit in mathematical performance (Devine et al., 2018). Impairments in developmental mathematical learning are termed as mathematical learning disability or developmental dyscalculia (Devine et al., 2018).

Didactic Materials: Hands-on materials used in Montessori education that supports sensory education and language (Montessori, 1964).

Early Childcare: Preschools, childcare centers, and family childcare homes (regulated and unregulated) (Kamerman & Gatenio-Gabel, 2007).

Early Intervention: Identification and servicing of children with and without disabilities in mathematics and/or reading. The term also refers to special education early intervention services for children ages 3 through 5 (Office of Special Education and Rehabilitative Services, 2016), domain specific interventions in preschool (DuPaul et al., 2015; Hinton et al., 2016; Khomais, 2014; Kroesbergen et al., 2014; Kyttala et al., 2015; Passolunghi & Costa, 2016), intervention strategies (Davenport & Johnston, 2015), and/or specific computer-based training programs (Mendizabal et al., 2015).

Informal Numeracy Skills: Informal numeracy skills are a set of skills used for basic number counting: *numbering* (verbal counting, counting forward and backwards, identification counting errors, structured counting, cardinality, resultative counting, counting a subset, subitizing, and estimation); *relations* (ordinality, relative size, number comparison, set comparison, number order, sequencing, number or set reproduction, number identification, and numeration); and *arithmetic operations* (adding and subtracting with and without objects, initial equivalence, equivalent sets, and number combinations) (Purpura & Lonigan, 2013).

Mathematics Manipulatives: Hands-on, concrete objects or tools needed for classroom instruction in mathematics (Carbonneau et al., 2013; Peterson & McNeil, 2013).

Montessori Method: An approach and methodology created by Maria Montessori that focuses on a child's natural physiological and psychical development using sensory and didactic materials to teach young children (Montessori, 1964).

One-to-One Correspondence: The ability to match numbers to objects and objects to objects; identifying numerically matching pair objects (Cakiroglu & Taskin, 2016).

Sequencing of Numbers: A fundamental principle of counting in which children learn the number names and number that appears in a fixed order (Carpenter et al., 2017).

Short Bead Stairs: A series of beads and wooden cards used for teaching counting, identifying numbers, identifying quantity, and identifying the order of numbers. These educational tools introduce place value in the decimal system to grasp the idea of quantity using a set of colored glass beads representing an appropriate number (Glermain, 2008; Montessori, 1964

Stable Order / Ordinality: Ordered sequencing of counting numbers in a collection in the same order (Carpenter et al., 2017; Marmasse et al., 2000; Purpura & Lonigan 2013; Reid, 2016).

CHAPTER II

LITERATURE REVIEW

Given that foundational mathematical concepts are developed at an early age, it would be valuable to fully understand the effects of the Montessori Methodology as it relates to early numeracy skills (Haq & Alfilfili, 2015; Laski et al., 2015; Lillard, 2012; Lillard & Else-Quest, 2006). Compared to research in reading, relatively limited research has been conducted to identify how children with developmental delays acquire number skills or other basic mathematical concepts (Cheung et al., 2017; Dolschield et al., 2017; Jimenez-Fernandez, 2015; Peterson & McNeil, 2013; Pinhas et al., 2014; Price, 2013; Purpura & Lonigan, 2013; Shusterman et al., 2016; Soydan, 2015; Van Herwegen, Costa, Nicholson, & Donlan, 2018; Xin et al., 2016). Correspondingly, there is a lack of empirical data to validate the use of Montessori materials in mathematical counting and number knowledge for preschool children (Kayili & Ari, 2011).

The research literature identifies that at an early age, mathematical skills are developed, future deficits can be identified, and early intervention of foundational mathematical concepts will reduce mathematical delays in later years (Bassok et al., 2016; Cimpian et al., 2016; Dunphy, Dolley, & Shield, 2014; Hannula-Sormunen et al., 2015; Hunting, 2013; Jordan & Levine, 2009; Lee & Md-Yunus, 2016; Purpura et al., 2015; Reardon & Portilla, 2016; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; West, 2017). It is, therefore, imperative that we identify appropriate evidence-based mathematical interventions for preschool-aged children (Carbonneau et al., 2013; Dunphy et al., 2014; Foreman & Gubbins, 2015; Hannula-Sormunen et al., 2015; Jacobi-Vessels, Brown, Molfese, & Do, 2016; Martin, Smith, Brasiel, & Sorensen, 2017;

Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013; Nyugen et al., 2016; Purpura & Lonigan, 2013; Schneider et al., 2017; Van Herwegen et al., 2018; West, 2017; Wolfgang et al., 2001; Zhu et al., 2017).

The primary objective of this chapter is to present an overview of the literature so as to provide the framework that children use to develop early numeracy skills through the utilization and implementation of diverse curricula and methods. This review provides specific knowledge about early numeracy skills of preschool children with and without developmental delays, as well as an overview of the differences between traditional and Montessori methods and curricula. This chapter is organized into eight major components that include: (1) the theoretical frameworks of Constructivism, Jean Piaget, and Maria Montessori; (2) early numeracy development including counting principles, sequencing of numbers, and basic number operations and mathematical understanding; (3) early childhood programming, data, and achievement; (4) special education and early intervention, including special education disabilities categories; (5) influence of teachers in a preschool classroom; (6) mathematical skill development of children, including influential factors for mathematical growth in children and mathematical skills development of children with disabilities; (7) mathematics curricula of traditional mathematics and the Montessori Method of mathematics; and (8) Montessori instruction for children with and without disabilities. The literature review concludes with a summary highlighting the significance of the study to professionals servicing preschool-aged children with and without developmental delays.

Theoretical Framework

Research in the Montessori field is limited to qualitative and descriptive studies primarily focused on curriculum for students; therefore, this study evaluated the integration of one traditional and one Montessori material that explicated curriculum in mathematics from a quantitative perspective. Beginning with a constructivist learning viewpoint, this study was guided by the theoretical and pivotal work of Jean Piaget and Maria Montessori. Piaget utilized a constructivist theory to clarify the process of learning, especially for children with disabilities (Brewer & Daane, 2002; Navarro, 2014; Roth 2017; Steffe, 2016; Xin et al., 2016). Both Piaget (1942) and Montessori (1967) developed theories that incorporated the use of concrete objects and a child's cognitive development (Capriora & Anghelide, 2016; Montessori, 1936; Montessori 1967; Pickering, 1992; Roth, 2017). As young children have difficulty thinking abstractly, these theorists argued that concrete materials should be used for young children through an experiential process (Capriora & Anghelide, 2016; McNeil & Uttal, 2009; Navarro, 2014; Steffe, 2016).

Special education literature focuses on behaviorally oriented pedagogy and less on the alternative theoretical orientations of Piagetian-oriented thinking (Gersten et al., 2009; Tzur et al., 2013). The common framework revolves around approaches such as explicit instruction and their effectiveness in providing interventions for students with disabilities (Gersten et al., 2009). Curriculum effectiveness is a necessary component to instructional methods. It can be argued that classical assumptions, such as explicit instruction, about effective curriculum pose issues because of the need to address student understanding, teaching conceptual understanding, and cultivating thinking specifically outlined in mathematical standards (Woodward & Tzur, 2017). The theoretical framework that is based on the work of Piaget and Montessori explained the basis for understanding numeracy skills and development for young children, which encompasses the following: (1) the foundations of mathematics; (2) constructivism; (3) Piaget's Theory of Cognitive Development; and (4) the Montessori Methodology.

Foundations of Mathematics

Mathematical foundations derived from earlier doctrines, such as constructivism, formalism, and Platonism (Damnjanovic, 2012; Dunphy et al., 2014). The view that mathematical objects are factual and objective, and that they exist solely on an individual's knowledge of them is derivative of Platonists (Dunphy et al., 2014). On the other hand, formalists believe that there are no mathematical objects and that mathematics is based on definitions and theorems (Damnjanovic, 2012; Davis & Hersh, 1981). Formalists focus on the rules and the connection between formulas. Conversely, the constructivist perspective is that objects are what individuals construct. Some argue that these views are limited, and other stances exist. Hersh (1997) adopted a more humanistic view on the foundation of mathematics, such that mathematics is innate and is a part of human culture, mathematical knowledge can be altered and corrected through trial and error, and mathematical objects are historic and distinct.

As the basis for this research focuses on the mathematical development of children, it is important to understand the foundations of mathematics based on prior dogmas and to explore the research of early numeracy development (Nguyen et al., 2016; West, 2017). Thus, mathematics is used every day and continues to be a major component of a child's development (Damnjanovic, 2012).

Constructivism

Mathematics can be challenging for learners because it requires a considerable effort on the part of the learner (Caprioara & Anghelide, 2016). More specifically, constructivist learning takes place through an exchange of views and problem solving (Brewer & Daane, 2002; Navarro, 2014; Roth, 2017; Steffe, 2016). Xin et al. (2016) examined how students with learning disabilities responded to constructivist-oriented mathematics instructions and concluded that children need more opportunities and prompting. In addition, more efforts in mathematics instruction are needed for students to understand abstract and conceptual problem solving. For the basis of this framework, the role of the teacher (Brooks & Brooks, 1999; Caprioara & Angehilde, 2016; Roth, 2017; Steffe, 2016), the students' engagement with mathematics (Roth, 2017; Steffe, 2016), and how numbers are processed (Cakiroglu & Taskin, 2016; Navarro, 2014) in a constructivist classroom were explored. Major components that lay the foundation for the constructivist theory and guide the theoretical framework of this study included: (1) the role of the teacher; (2) students' understanding of mathematics; and (3) conservation of numbers.

The role of the teacher. In a constructivist classroom, the student takes responsibility for his/her own learning (Brooks & Brooks, 1999). As a teacher, the responsibility to create an educational environment for the student to assume his/her responsibility for learning is key. By creating an educational environment in a constructivist classroom, teachers provide appropriate materials for learning, encourage self-initiation, and perceptively interact with students (Brooks & Brooks, 1999). With regard to the student's engagement with mathematics, research on constructivism also forms distinctions related to the teacher. Roth (2017) conducted a study on the importance of 'teaching for astonishment' as children can learn to appreciate something new in their learning and development process. The goal of constructivist teaching is to allow a child to be a part of the changes that occur or interact with mathematical learning to guide the teacher's directives (Steffe, 2016). The teacher, therefore, serves as a facilitator, guide, supporter, organizer, and coordinator (Caprioara & Angehilde, 2016).

Mathematics of the student. In education, constructivist learning theories have become some of the most dominant ideologies because of the focus on the identity of an individual through the learning and development process (Roth, 2017). Steffe (2016) described these individual learning processes as the "mathematics of the student." The mathematics of the student was characterized by the student's ways of operating that demonstrated his/her engagement in mathematics activity and interactivity (Steffe, 2016). There is a clear indication between the detailed accounts of mathematics of the student and experiential models of learning, such as when a student is actively engaged in mathematical learning (Steffe, 2016). Children use their prior experiences to actively engage in mathematics. Their understanding of mathematics began before they were formally introduced to numbers and quantity; however, their ability to express their knowledge begins once the child begins to speak (Cheung et al., 2017). At this point, children begin to generalize their knowledge after learning to count. Therefore, mathematical knowledge is built on the foundations set by experiential learning of the student (Caprioara & Anghelide, 2016). Having a strong foundation of experiential learning and prior knowledge is needed for further mathematics skills.

Conservation of numbers. According to the constructivist theory of learning, small children learn number concepts by initially being taught the logical processes and how they are organized (Navarro, 2014). The theory was designed with a set of procedures that studied the acquisition of numeracy, such as seriation, conservation, and correspondence. Research has enhanced these processes by concluding that when children ages three through six are presented with non-verbal conditions, their numeracy abilities improve dramatically (Navarro, 2014); however, gender has not yet served as an indicator of developing number concept acquisition (Cakiroglu & Taskin, 2016).

Piaget's Constructivist Theory of Learning – Theory of Cognitive Development

Mathematical concepts are scaffolded to build off one another, much like the theory of cognitive development (Papadakis, Kalogiannakis, & Zaranis, 2017). Jean Piaget was an influential constructivist who was concerned with cognitive development and the formation of knowledge (Brooks & Brooks, 1999). In 1936, Piaget concentrated on the learning process and how mathematics was learned. He was concerned with the developmental stages of a child's cognition. His constructivist theory states that learning is active, direct, and a practical process that is managed, assessed, and initiated by the learner (Caprioara & Anghelide, 2016). He proposed that a child's cognition develops through uninterrupted conversions of thought processes (Ghazi & Ullah, 2015; Ghazi & Ullah, 2016). According to Lefmann and Combs-Orme (2013), Piaget proposed that a child's experiences in one stage forms the foundation for the next. Although Piaget proposed that children go through the same sequence of development, he also proposed that time spent in each stage varies (Lefa, 2014).

Based on constructivist ideals in which an individual's experiences, prior and new knowledge is constructed for learning, Piaget proposed that individuals would pass through these stages of development at their own pace (Brooks & Brooks, 1999). Children, in this case, are an integral part of the learning process and must be active participants to understand what he/she already knows before the teacher presents new information (Ghazi & Ullah, 2015; Ghazi & Ullah, 2016; Reedal, 2010). Piaget's theory of learning was most related to the mathematical concepts of comparing numbers and one-to-one correspondence and the long-term effects of these concepts on the development of young children's learning motivation (Hu, Jia, Plucker, & Shan, 2016). Piaget's Theory of Cognitive Development identified four primary stages of development that all children progress through at different times of their lives: (1) sensorimotor, (2) preoperational, (3) concrete operational, and (4) formal operational.

Sensorimotor stage. In the sensorimotor stage, Piaget proposed that during infancy (birth to the presence of language - 2 years old), cognitive and mental characteristics are developed (Caprioara & Angehilde, 2016; Kose & Arslan, 2015; Lefmann & Combs-Orme, 2013; Reedal, 2010). At this stage, he proposed that infants have the ability to find objects subsequent to displacement, link numbers to objects, and develop basic understanding of the concepts of numbers and counting (Kose & Arslan, 2015). During this stage, children utilize their five senses and concrete experiences to understand mathematical concepts. More specifically, children learn object permanence in the sensorimotor stage in which they are able to find objects that have been removed from view (Capriora & Anghelide, 2016; Lefmann & Combs-Orme, 2013). Children also develop an understanding that an object once in plain sight, can be removed and still exists even though they are unable to see it.

Early numeracy skill development difficulties can be identified at this stage as well (Hannula-Sormunen et al., 2015; Lee & Md-Yunus, 2016; Purpura et al., 2015). Carpenter et al. (2017) suggested that infants in this stage have some understanding of counting and the concept of numbers. Children also begin using their five senses to develop number awareness (Lefmann & Combs-Orme, 2013). They use their fingers to demonstrate their age or use concrete objects to demonstrate counting. Children demonstrate their ability to identify numbers in a one-to-one correspondence in which each number can be represented by a concrete object (Carpenter et al., 2017; Hu et al., 2016; Kose & Aslan, 2015; Marmasse et al, 2000; Reedal, 2010). One-to-one correspondence is presented at this stage as children have the ability to match objects to objects, and numbers to objects (Kose & Aslan, 2015; Reedal, 2010). However, although children struggling with one-to-one correspondence are able to recite the numbers one through ten, they are unable to match numbers to objects.

Preoperational stage. During the preoperational stage (two to seven years old), language and symbolic abilities are increased as children have the capability to verbalize the thought process (Capriora & Anghelide, 2016; Lefa, 2014; Lefmann & Combs-Orme, 2013; Navarro, 2014; Steffe, 2016; Tunyiova & Sarmany-Schuller, 2016). Logic is limited at the preoperational stage and children's ability to identify objects are restricted to one dimension; however, children at this stage have the ability to complete one-step logic problems and develop language (Haq & Alfilifili, 2015). Mathematical language, logic problem-solving, and operational completion is also developed during this stage

(Haq & Alfilfili, 2015; Reedal, 2010). Similarly, children continue to incorporate concrete objects for counting; yet, rational and logical thinking is limited. Additonally, squential order, basic mathematical operations such as addition and subtraction, and the elaboration of one-to-one correspondence is developed (Carpenter et al., 2017; Hu et al., 2016; Kose & Aslan, 2015; Reedal, 2010).

Research supports numeracy abilities in the preoperational stage for both symbolic and non-symbolic learning (Schneider, et al., 2017; Van Herwegen et al., 2018; Zhu et al., 2017). In a comparative study of forty-nine preschool children with low mathematical abilities and twenty preschoolers with no identified mathematical difficulties, results proved that ordinality knowledge is an important factor of mathematical development as students improved on their mathematical abilities using a symbolic or non-symbolic intervention program (Van Herwegen et al., 2018). Similarly, students who were provided the non-symbolic interventions improved on the Approximate Number Sense task, counting tasks, and digit recognition tasks and maintained improvements six months later when compared to those who were provided solely the symbolic intervention (Van Herwegen et al., 2018). Moreover, a meta-analysis on symbolic and non-symbolic numerical magnitude processing proved better results in preschool children (Schneider et al., 2017).

Concrete operational and formal stage. Subsequent to the first two stages, Piaget noted that the concrete operations and formal operations stages are a continuation of the beginning two stages, with more complexities and abstract mathematical problems (Ghazi & Ullah, 2015; Hu et al., 2016; Lefmann & Combs-Orme, 2013; Leongson & Limpjap, 2003). Although the stages of cognitive development are mostly focused on the childhood years, the final stage is said to continue through adulthood (Steffe, 2016). In Leongson and Limjap (2003), there were over 50% of college freshmen identified in the concrete operations stage of Piaget's theory of development when it was presumed that all students at the college were in the final stage of development. This study was a clear indication of the importance of not assuming a child's capability of understanding without initially assessing the child's cognitive ability. In Ghazi and Ullah (2016), the application of the Concrete Operational Stage of Piaget's Cognitive Development Theory was used to compare academic achievement of 200 males, aged 7-11, from urban and rural areas. The results indicated that although students can understand the conservation of numbers and ordering, logic is more successful (Ghazi & Ullah, 2016).

Cognitive growth emerges at the concrete operational stage (7-11 years old) when a child's language and basic skills are developed (Capriora & Anghelide, 2016; Navarro, 2014; Papadakis et al., 2017). Two essential components needed for understanding number concepts that are developed in this stage are logical operations of seriation and classification (Ghazi & Ullah, 2015; Ghazi & Ullam, 2016; Tunyiova & Sarmany-Schuller, 2016). Children have the ability to form hypotheses and deduce potential consequences in the formal operational stage (11-16 years old) (Hu et al., 2016; Lefa, 2014; Tunyiova & Sarmany-Schuller, 2016). The development of abstract reasoning without concrete representations is probable (Tunyiova & Sarmany-Schuller, 2016); reasoning skills include: clarification, inference, evaluation, and application. Thus, the evolution and inclusion of prior stages of a child's development is needed for more complex reasoning to take place.

Theory in practice. Piaget's theory of cognitive development has been utilized in a variety of academic disciplines; however, it is especially applicable in mathematics (Ghazi & Ullah, 2015; Ghazi & Ullam, 2016; Reedal, 2010; Tunyiova & Sarmany-Schuller, 2016). His work concluded that "the growth of knowledge is the result of individual constructions made by the learner" (Brooks & Brooks, 1999, p. 25). As children progress through the four stages of cognitive development according to Piaget, (1) hand-eye coordination and object permanence are developed (sensorimotor), (2) symbolic thought and language progress (preoperational), (3) basic operations such as classification and seriation are performed (concrete operational), and (4) abstract thinking are developed (formal operational) (Hu et al., 2016; Lefmann & Combs-Orme, 2013). As such, Piaget's first two stages of cognitive development were explored for the purpose of this study with emphasis on constructivist learning, which focuses on the process of learning.

Montessori Approach

As an approach to learning, Maria Montessori (1870-1952) developed the Montessori Method that emphasized active learning, child independence, cooperation and collaboration, and learning at a child's individual pace of development (Montessori, 1964. Her views were based on what she called "normalization." Montessori proposed that children "normalized" when they had the ability to reach a sense of autonomy and self-governance when provided with the right environment (Frierson, 2016). In her methodology, Montessori explained how an absence in the environment and other external conditions guided a child's autonomy, and it was not solely based on the limitations of the stage of his/her life development (Frierson, 2016).

Montessori proposed that children are naturally curious and driven to learn; therefore, learning should be viewed as a process (Montessori, 1936; Pickering, 1992). These thought processes emphasized the unconscious cognitive processes and their relationship on early childhood education (Frierson, 2014). Her approach focused on all aspects of development rather than specific components. More specifically, the Montessori Methodology focused on (1) respect for the child, (2) the absorbent mind, (3) sensitive periods, (4) the prepared environment, (5) planes of development, (6) three period lesson, (7) the teacher's role, and (8) the *Children's House* (Montessori, 1936; Montessori, 1967; Pickering 1992).

Respect for the child. In today's society, too often respect for children is limited to higher expectations and enforced discipline. Montessori (1967) described how adults lacked respect for children by forcing children to follow rules without attention to their individual needs, being overbearing and rude, and expecting them to be well-behaved at all times. As Montessori observed children, she noticed how children imitated adults and, therefore, should treat children in ways that will foster the behaviors we would want them to develop (Montessori, 1967).

In the Montessori approach, children are offered choices that prepare them to become independent learners. Instead of being told what to learn, children are embraced with hands-on learning from the world around them (Montessori, 1967). Respecting the child in a variety of ways allows the child to explore learning, which drives motivation. Teachers also have the ability to show respect for a child by modeling lessons and activities and teaching them to learn for themselves.

The absorbent mind. Dr. Maria Montessori developed the Montessori Method during the early twentieth century specifically for children with intellectual and developmental disabilities (then referred to as mental retardation) (Montessori, 1936; Montessori, 1967). Her work focused on creating a "prepared environment" (an environment with materials formulated and specifically designed for young children) in which children needed less teacher direction and more engagement with sensorial materials (Montessori, 1936; Pickering, 1992). During this time, she allowed the children to interact with the prepared environment and used what she considered a child's absorbent mind. As a result, Montessori proposed that children's minds have the ability and capacity to take in information and develop based on their experiences and surroundings. She proposed that teachers serve as facilitators to help children do things on their own. Although it is not the sole predictor of development, Manan and Khadija-Tul-Kubra (2017) proposed that the younger the child, the more absorbent is his/her mind. Montessori (1964) suggested that a natural assumption is that prior knowledge is the foundation for the "first stages of essential culture – writing, reading, and number, and that knowledge comes as an easy, spontaneous, and logical consequence of the preparation" (p.84).

Sensitive periods. According to Frierson's (2014) rendition of Montessori's epistemology, he noted how children depend on their interests to build experiences. Any experience that goes beyond the interest of a child, motivation is needed to continue (Frierson, 2014). Montessori (1964) identified "these powers to be innate predispositions that manifest into 'sensitive periods' of interest (p.188)." A child's interest drives these sense experiences: "In the world around us, we do not see everything… but only some things that suit us…We do not concentrate our attention haphazardly… but according to an inner drive" (Montessori, 1964, p. 185).

Montessori also proposed that children had "sensitive periods" in which they had the ability to retain and learn new skills (Montessori, 1967). She concluded that children learned at different times and at their own unique pace; thus, it is important to allow a child to lead his/her choice of activities. Therefore, teachers are responsible for understanding the appropriate time to introduce a new concept to an individual.

The prepared environment. A Montessori classroom is physically organized with learning materials that are readily available for the child, known as *didactic materials*. Didactic materials are instructional materials designed or intended to teach (Frierson, 2014). These materials in the classroom are utilized for a child's learning and experiences to take place. Montessori designed these didactic materials to offer a 'sensory education' (Montessori, 1964). Based on her own observations, Montessori (1964) claimed that children can repeat exercises with these didactic materials over forty times (p.32). Repetition of lessons are important for child learning and experiences in a Montessori education.

Zadnik and Koren (2017) conducted a five-month project on the correlation between Montessori pedagogy and music. They indicated that the prepared learning materials used in Montessori were attractive to children, increased their motivation for learning, and had impact on building the child's autonomy. More specifically, the materials in a Montessori environment vary based on differing age groups, characteristics, and interests for the variety of children in the classroom. In a prepared environment, Lillard (2013) discussed how Montessori education is considered a form of playful learning in which children use hands-on materials to enhance learning in the classroom. Many preschool teachers today adopt the learning environment from the Montessori approach (Soydan, 2015).

Planes of development. Montessori's multi-sensory approach has been the foundation for many other perspectives. Montessori (1931) expounds on the process of "developing the mathematical mind" in this sensory and material-based learning. As Maria Montessori explained that children learn and develop in different phases, known as *Planes of Development*, Pickering (1992) explains that children in the first plane (ages birth to 6 years) are more attentive to the organization and classification of objects, and children in the second plane (ages 7-12) are more attentive to organization and classification of abstractions. For the purpose of this study, the focus was on the first plane of development in which children are most attentive to organization and classification of early numeracy skills.

Infancy (birth-6 years). During the infancy stage, the absorbent mind and sensitive periods work together to intensify and bring about learning (Montessori, 1936). As children learn through their senses in the first three years of their life, they are then prepared to consciously use hands-on experiences to guide the next three years of their life (Montessori, 1967). Learning takes place as the individual child is prepared and allowed to do so on his/her own.

Childhood (6-12 years). During the childhood stage of the phases of development, children have the basic skills needed for learning. They are no longer in need of an absorbent mind and begin learning through cognitive reasoning and utilizing their imagination (Montessori, 1936; Montessori, 1967). As a result, children strive to learn about the world around them and factual information. Montessori did not develop a practical learning system beyond the childhood stage; however, she did identify the phases of development that succeeded childhood.

Adolescence (12-18 years). Subsequent to the phases of childhood, Montessori described adolescents. Adolescents in this phase of development are more aware with the connection between learning and their daily life skills. Montessori proposed that adolescents would be better prepared to adapt to the world by becoming independent learners based on survival skills and the natural world (Pickering, 1992).

Transition to adulthood (18-24 years). The final phase of development focused on career exploration and the beginning of adulthood. With the necessary cognitive and social skills development from prior phases, transition to adulthood would encourage a young adult to make satisfying career choices (Montessori, 1964).

Three period lesson. During the stages of learning, Montessori proposed children learn through to a "three period lesson." The three-period lesson is used to introduce and evaluate a child's mastery of a concept (Gitter, 1967). In the first period, the association of sensory perception with its name takes place (e.g. "This is __________."). During the second period, a child has the ability to recognize an object corresponding to the name (e.g. "give me ____ or show me ___"). Lastly, the third period is when a child remembers the name corresponding to the object (e.g. "What is this?"). As a result, this evaluation identifies a child's mastery of numbers or counting. This specific technique is most helpful for students with disabilities because concrete manipulatives and materials are used, not abstractions (Gitter, 1967). Similarly, modeling, repetition, and exploration also take place during the three-period lesson. The learning process provides a foundation for the student, not the teacher, to notice mistakes and make corrections. Montessori (1967) admired the child's attainment of knowledge and his/her ability to self-correct that she identified this as the process of auto-education.

The teacher's role. Similar to Piaget's constructivist theory, Montessori also proposed that a teacher's role was not that of classical educational methods. Montessori proposed that a teacher's role was that of an organizer, facilitator, observer, mediator, supporter, and collaborator with the child (Ivanova, 2014). In fact, Montessori proposed that teachers specifically served as facilitators in the classroom (Barbieru, 2016; Montessori, 1936; Pickering, 1992). More specifically, the role of the teacher is to aid the child's learning process, strategically plan and prepare the classroom with appropriate and organized materials and allow the child to explore the classroom while observing the child. The teacher also offers guidance and assesses the child's sensitive periods and need for the introduction of new concepts (Barbieuri, 2016; Montessori, 1936; Montessori, 1967).

Montessori's belief about a teachers' role in the classroom is critical for the methodology. Unlike traditional education, the Montessori teacher introduces a new skill and removes himself/herself to observe (Barbieuri, 2016). Based on Montessori's understanding of a child's *auto-education* (the process in which a child leads him/herself to self-correct), children are able to train themselves to observe, make comparisons, form judgments, and repeat exercises indefinitely (Montessori, 1964).

Initially, the teacher demonstrates the exercise to the student first, showing the child how pieces should be arranged (Montessori, 1964). Subsequently, the teacher then observes the child directly and indirectly while in the classroom. The teacher's role is not to prevent error but to prevent "rough or disorderly use of the material" (Montessori, 1964, p.35). The child's repetition of the exercise is what children embrace the most and teaches the child to correct him/herself. When the child practices the exercise repeatedly, the teacher does not need to intervene. However, when a child makes a mistake, it is understood that the child has not yet reached the stage of understanding (Montessori, 1964). It is essentially important for the teacher to guide the child without letting the child know of her presence. Thus, this is important for the teacher to provide support when needed yet does not interfere with the child learning from his/her experiences (Montessori, 1964).

The Children's House – Casa dei Bambini. Based on the foundations and principles of the Montessori Method, Maria Montessori designed and named the child's environment, *The Children's House.* This "laboratory which will bring more truth to light than thus hitherto recognized" allows children to freely work and develop (Montessori, 1964, p. 186). This setting is the ideal setup of a Montessori classroom in which children have the ability to apply their skills and learned behaviors. The Children's House is also the environment in which the teacher has the ability to serve as the facilitator and model lessons for children, allowing children to self-correct and complete tasks independently. Without the specific structure of the Children's House, the Montessori environment is nothing more than a classroom with hands-on materials. It is important for the environment to not only possess hands-on materials, but to also include: special work environments (easily accessible materials on child-sized shelves, rugs for working on the floor, mats for working at tables, etc.); a child-centered classroom in which the teacher serves as the facilitator and works individually with students versus whole group; and meaningful lessons that are sequentially designed for children's learning (Pickering, 1992).

Summary of Theoretical Framework

The theoretical framework for this study focused on the foundations of mathematics and its direct correlation with constructivist perspectives. Constructivists believe that mathematical objects are factual and objective (Dunphy et al., 2014) and that mathematics is a major component of a child's development (Damnjanovic, 2012). For the basis of this framework, the role of the teacher (Brooks & Brooks, 1999; Caprioara & Angehilde, 2016; Roth, 2017; Steffe, 2016), the students' engagement with mathematics (Roth, 2017; Steffe, 2016), and how numbers are processed (Cakiroglu & Taskin, 2016; Navarro, 2014) in a constructivist classroom were explored. This study highlighted: (a) the role of the teacher in the classroom as the individual who creates an educational environment for the student to assume responsibility for learning (Brooks & Brooks, 1999), (b) the importance of experiential learning for the development of mathematical knowledge for children (Carprioara & Anghelide, 2016), and (c) the constructivist view on numbers as children learn concepts by being initially taught logical processes and how they are organized (Navarro, 2014).

As mathematical concepts are scaffolded to build off one another, Piaget's Theory of Cognitive Development served as the overarching theory, focusing on the learning process and how mathematics is learned (Papadakis et al., 2017). Piaget's theory includes four stages; however, this study focused on the second stage of development: the preoperational stage for children two to seven years of age. During this stage, children increase their language and symbolic abilities (Capriora & Anghelide, 2016; Lefa, 2014;

Lefmann & Combs-Orme, 2013; Navarro, 2014; Steffe, 2016; Tuniyiova & Sarmany-Schuller, 2016), have the ability to verbalize their thought processes and complete one-step logic problems (Haq & Alfilfili, 2015; Reedal, 2010), develop mathematical language, logic, and problem solving (Haq & Alfilfili, 2015; Reedal, 2010), and continue to incorporate concrete objects for counting.

The second level of Piaget's theory that was focused on in this study was the integration of the integration of Montessori's concept of the Planes of Development and specifically the use of the Montessori Short Bead Stairs. In a multi-sensory approach, Maria Montessori proposed that children were independent learners who cooperated and collaborated in a classroom environment at a child's pace (Montessori, 1967). This study highlighted the major components of how children learn according to Montessori: children embrace hands-on learning from the world around them; children need less teacher direction and use their absorbent mind to engage with sensorial materials in a prepared environment; and children use their ability to retain and learn new skills during their sensitive period (Montessori, 1967; Pickering 1992). This multi-sensory approach was used to develop a mathematical mind in which sensory and material-based learning occurred through the Planes of Development. This study incorporated a prepared environment with didactic materials for children in the first plane of development. During this plane children were most attentive to the organization and classification of objects; and through a three-period lesson, this study introduced new concepts for preschool children to learn. As a result, the teacher served as a facilitator and supporter for the basis of this study (Ivanova, 2014).

The Development of Early Numeracy

Most children have the capacity to develop appropriate mathematical foundations for future mathematical abilities that are innate and develop during the earlier stages of life (Hannula-Sormunen et al., 2015; Jordan & Levine, 2009; Purpura et al., 2015). Initially, Piaget proposed that babies are born without understanding of numerosity and the ability to discriminate object quantity (Piaget, 1942); however, recent studies have shown how infants are able to discriminate the numerosity of two to three objects (Feigenson & Carey, 2005; Green, Gallaghar, & Hart, 2018; Huntley-Fenner, 2001).

A number of studies have demonstrated how individuals have developed varying degrees of mathematical knowledge by the age of five (Huntley-Fenner, 2001; Gallistel & Gelman, 2000; Varol & Farran, 2006). Interestingly, within the first few weeks of life, an infant has the ability to notice the smaller numbers (Antell & Keating, 1983; Feigenson & Carey, 2005). By the age of 18 months, infants have the ability to gradually develop awareness for smaller values of numbers (Geary, 1994). Thus, infants are sensitive to small changes in numbers, such as adding 1 and 1. Consequently, by the age of two, children begin learning to count without the ability to decipher cardinal numbers until the age of four or five (Maxim, 1989).

Young children also have the capacity to identify number sets up to five without counting (Hunting, 2013). Zur and Gelman (2004) state that 3-year-old children are able to count and use basic concepts for predicting and checking addition problems for problem sets up to five items. Children also have the ability to learn number words, such as "one, two, three...", and combine them with numerical concepts (Marmasse et al., 2000). The ability to understand numerosity during the first few months and years of life indicate an innate ability to develop a strong foundation for numerical skills and capacities (Carpenter et al., 2017; Marmasse et al., 2000).

Research highlights the importance of instinctive knowledge and not teacher instruction on counting skills (Feza, 2016). In a qualitative analysis, Feza (2016) indicated that teacher knowledge was not essential for teaching counting for five and six-year-old children in Africa. Further analysis stated that the development of counting skills was inconclusive to educators (Feza, 2016).

Therefore, early mathematical skills are influenced by the child's ability to keep number words in short-term memory, which are needed for counting (Carpenter et al., 2017; Marmasse et al., 2000). However, understanding basic number counting, does not correlate to understanding the principles of numeracy nor does it drive the cardinal principle induction (Cheung et al., 2017). In fact, Montessori (1964) addressed "instinctive knowledge" as an imperative basis for understanding numeration. Specifically, as it related to 'quantity', all Montessori materials were designed with this idea in mind. Materials were designed as longer or shorter, and/or darker or lighter to elaborate the meaning of quantity and the presence of the human senses in Montessori materials (Montessori, 1964).

Early numerical skills are important for success in mathematics later in life (Dunphy et al., 2014; Hannula-Sormunen et al., 2015; Jacobi-Vessels et al., 2016; Nguyen et al., 2016; West, 2017). As a result, Nguyen et al. (2016) conducted a study using longitudinal data from a sample of children that examined the extent of their preschool mathematical competencies, such as counting, and its prediction on fifth grade mathematics achievement. The study found that early numeracy abilities were among the strongest predictors of later mathematics achievement; thus, highlighting the importance of preschool mathematics knowledge on future achievement (Nguyen et al., 2016). Hannula-Sormunen et al. (2015) investigated 36 children diagnosed with neurological disorders who participated in a seven-year longitudinal study that examined how their spontaneous focusing on numerosity, subitizing based enumeration, and counting skills at 5 and 6 years of age predicted their mathematics achievement at 12 years old. The results demonstrated that verbal counting skills and spontaneous focusing on numerosity before kindergarten predicted their mathematical performance in fifth grade. Similarly, West (2017) analyzed two national longitudinal kindergarten cohort studies that were conducted over a 3-year period based on the Early Childhood Longitudinal Study (ECLS) program. Results from the *ECLS – K* and *ECLS – 2011* indicated a number of conclusions related to overall performance in mathematics. West (2017) concluded that children made minor gains in kindergarten mathematics assessments but demonstrated larger gains in subsequent years.

Furthermore, research on numerical knowledge established insight into young children's mathematical understanding and capabilities (Aunio et al., 2014; Sophian, 2009). A fundamental component of numerical knowledge relates to a child's ability to compare sets of numbers. Sophian (2009) suggests that the ability to identify less-than, greater-than, and equal relationships between sets of numbers is a significant achievement for preschool children. Although children demonstrated the ability to compare sets of similar objects, they struggled to compare sets of numbers with contrasting objects (e.g., lion figurines compared to dots) (Sophian, 2009). Aunio et al.

(2014) investigated the similarities and differences between children's early numeracy skills and their relation to age, nationality, and gender. Five to seven-year-olds from Finland and Iran were assessed by using number-related relational skills tasks and counting skills (Aunio et al., 2014). Differences between gender, age, and nationality existed; thus, indicating that mathematical understanding varies in different parts of the world and for different groups.

As number knowledge in preschoolers has been the focus of past mathematical research, further research identified preschool engagement of algebraic thinking, specifically patterning (Rittle-Johnson et al., 2013). Rittle-Johnson et al. (2013) assessed the repeating pattern knowledge of 68 four-year-old children on two separate occasions. As children were able to extend or duplicate patterns, some demonstrated an explicit knowledge of pattern units. Additionally, pattern knowledge in some areas were evident before children were successful on specific items; thus, indicating that children develop an understanding of repeating patterns before kindergarten (Rittle-Johnson et al., 2013).

Early numeracy development is based on various components. The following components are related to this study and explained the research supporting or invalidating the importance of early numeracy skills: (1) counting principles; (2) one-to-one correspondence; (3) stable order; (4) cardinality; (5) sequencing of numbers; and (6) basic number operations and mathematical understanding, including the base-ten number system, counting 1-20, spoken and written numbers, and informal numeracy skills.

Counting Principles

One of the main goals in early childhood education is for children to develop basic number concepts (Carpenter et al., 2017). Counting and learning to count is one of the earliest forms of mathematical activities children engage in and serves as the basis for number concepts and skills (Carpenter et al., 2017). Learning to count involves reciting sequential numbers and using those sequences of number names to count a collection of objects, items, movements, or events. Thus, early in education, children need to be provided with structured activities that support their connections between counting and number structure (Voustina, 2016). As such, there are three major principles that define counting: one-to-one correspondence; ordered sequence of counting numbers; and cardinal order (Carpenter et al., 2017; Marmasse et al., 2000; Reid, 2016). Stable order and base-ten systems are also considered to be major principles of learning to count (Reid, 2016).

One-to-one correspondence. One-to-one correspondence refers to a counting sequence. Each item in the sequence is assigned exactly one number (Carpenter et al., 2017; Reid, 2016). For example, children use counting bears to count: one yellow bear (1), one green bear (2), and one red bear (3). The one-to-one correspondence principle states that there must be a one-to-one correspondence between numbers so that objects are only counted once (Carpenter et al., 2017; Marmasse et al., 2000). Children must keep track of which objects are counted to make sure there is an accurate one-to-one match in the collection. Most children often place objects in a straight line or in a specific type of group to make the one-to-one correspondence easier to identify and accurately count (Carpenter et al., 2017; Reid, 2016). However, even by lining up the objects, some children have not mastered the one-to-one correspondence principle and either skips objects or numbers in the sequence. One-to-one correspondence assumes that a child could place a finger over one object and count that object as 1, followed by a second object as 2, and so forth.

One-to-one correspondence for young children can be a difficult task (Izard et al., 2014). One study tested 2-year-old children with no knowledge of numbers beyond four. The children in this study were given one-to-one correspondence cues to track a set of five or six items and assessed by a non-verbal manual task; however, children failed to track and identify numbers when one element was added or removed (Izard et al., 2014). Although it can be difficult, research supports children's ability to use one-to-one correspondence in counting (Brueggemann & Gable, 2018; Green et al., 2018). Brueggemann and Gable (2018) conducted a study to investigate preschool children's selective sustained attention on early numeracy skills and development. The results indicated that young children had the ability to reliably count and learn that symbols represent quantities. A quasi-experimental study of 50 preschool children with disabilities investigated the effects of an interactive shared storybook reading intervention with mathematical content and activities on early numeracy skills (Green et al., 2018). The intervention included counting objects in the storybook, one-to-one correspondence, and comparing numbers. Findings concluded that children who received the intervention scored significantly higher than children who did not receive the intervention in total math ability, quality comparison, one-to-one correspondence counting, and oral counting as measured on the Test of Early Mathematics Ability, Third Edition (Green et al., 2018).

Stable order. Ordered sequencing of counting numbers refers to the order of numbers in a collection (Carpenter et al., 2017; Reid, 2016). As children learn to count, they begin with number one and continue in an increasing number order. These numbers are always assigned to items in a collection in the same order (Carpenter et al., 2017; Purpura & Lonigan, 2013). The awareness of ordering relationships between numbers is often referred to as *ordinality* (Marmasse et al., 2000). Ordinality is developed gradually across three to four values over the course of the first 18 months of a child's life (Geary, 1994).

Cardinality. Cardinal number principles refer to the last number in the counting sequence. A child's understanding that the last number word of a set of counted objects has meaning reflects the numerosity of the set of numbers (Marmasse et al., 2000; Reid, 2016). Once a child counts all of the objects in the collection, the last number in the counting sequence represents the entire collection (i.e., 1, 2, 3……10.). The goal of the cardinal number principle is to count a group of objects and find the total number. This principle is established after the one-to-one and stable-order principles are established because it is developed after a child has experience in selecting specific numbers and applying the numbers to a sequential set (Marmasse et al., 2000; Purpura & Lonigan, 2013).

Although number skills and cardinal number knowledge are linked together, they are not exclusively related (Dolschild et al., 2017; Shusterman et al., 2016). Dolschield et al. (2017) conducted a study to determine the correlation between a child's comprehension of numbers and their cardinal number knowledge in children. Results found that number knowledge does not support number acquisition; therefore, children understand cardinal numbers in a general way and not solely based on their understanding of the quantity of numbers (Dolschield et al., 2017). Additional research supports that cardinal numbers are not related to the verbal acquisition of numbers (Posid

& Cordes, 2015; Shusterman et al., 2016). In a longitudinal study of forty-six preschool children, Shusterman et al. (2016) identified a strong correlation between non-verbal number knowledge and the cardinal principle based on pre- and post-assessments of number acuity.

Further research discounts the relationship between spatial preferences and cardinality comprehension (Knudsen, Fischer, & Aschersleben, 2015). In a study of 104 preschool children and 182 parents, handedness, spatial tasks, number tasks, task order, and parental finger counting were assessed to determine the directional preferences of finger counting, object counting, and picture naming of preschool children (Knudsen et al., 2015). As the results indicated, a right-sided preference for finger counting and left-sided preference for counting and naming objects, children were consistent in their preference for counting and naming objects but not when pairing objects together. The overall results indicated that there was no relation to children learning the cardinality of numbers based on their spatial preferences.

Sequencing of Numbers

Children often begin learning to count by reciting a number sequence. One of the most fundamental principles of counting is counting the numbers in a fixed sequence (Carpenter et al., 2017). By doing so, children learn the number names and that number appears in a fixed order. Children learn the specific order of the numbers and that numbers cannot be repeated. The order of learning the sequence of numbers varies for each child. Some children learn number names, but do not understand there is a sequence to follow (Carpenter et al., 2017).

Basic Number Operations and Mathematical Understanding

Through the process of discovery, young children have the ability to develop number and basic number operational skills and other mathematical concepts (Brendefur, Strother, Thiede, Lange, & Surges-Prokop, 2013; Varol & Farran, 2006). For example, children use blocks in preschool and at home to classify, measure, count, order, and sort (Hewitt, 2001; Wolfgang et al., 2001). Often times, children may not know that they are engaging in mathematical conversation and play, but it becomes an integral part of their day (Carpenter et al., 2017). Similarly, teachers engage students in building math understanding through daily routines, informal play, and structured lessons. As the development of mathematical reasoning and understanding begins early in a human's life, it is important to foster and provide challenging skill development at an early age. Brendefur et al. (2013) conducted a study to examine the effects on 4-year-old's knowledge of mathematics by introducing professional development and mathematical interventions in four mathematical domains over a six-month period. Results found that children who received the intervention were more fluent and flexible with number concepts and solving problems than children without the professional development and mathematical interventions (Brendefur et al., 2013). Basic fundamental components and knowledge are needed for the development of mathematical reasoning, such as: (1) the base-ten number system. (2) counting 1-20, (3) spoken and written number knowledge, and (4) informal numeracy skills.

Base-ten number system. More specifically, learning to count requires a child to memorize a small sequence of number names that will eventually introduce larger number names (Carpenter et al., 2017). This system is referred to as the base-ten number system. The base-ten number system refers to the grouping of objects into groups of 10.

Once objects are counted to ten, tens can then be grouped, and remaining numbers are counted. Groups of tens can be grouped into hundreds, thousands, and so on.

Counting 1-20. In English, children must learn to memorize the numbers for 1 to 12 first. Upon mastery, the teen numbers (13-19) are learned. The teen numbers are rooted from the numbers three to nine but differ in language and structure. *Fourteen, fifteen, sixteen, seventeen, eighteen,* and *nineteen*, all have the root number plus the word "teen". Numbers such as *eleven, twelve,* and *thirteen* become complicated for younger children because there is no structure. Understanding the numbers 1-20 are based on memorization because learning the sequence is easier than learning the structure of the numbers (Carpenter et al., 2017). After 20, numbers begin to make sense and general rules can be applied. Children use their foundation of number one to nine to continue counting numbers beyond 20, including decade numbers (20, 30, 40, etc.). Although there is a set pattern of the number names and order, the mathematical language is not clear for children and can make learning and mastering numerosity difficult for some (Carpenter et al., 2017). Therefore, mastery of one to nine is essential for subsequent number acquisition.

Mastering the ability to count and make quantitative comparisons is essential for mathematics effectiveness later in life (Lee & Md-Yunus, 2016). One study investigated children's abilities to count and make quantitative comparisons on 34 preschool children via a clinical interview method through rote counting, rational counting with the cardinality rule, "zero" concept, and quantity comparison between sets of blocks (Lee & Md-Yunus, 2016). Results of the study indicated that 89% of preschool children were able to do rote counting and used a counting strategy, 70% were able to do rational counting, and 65% of the participants understood quantitative comparisons of objects.

In an experimental study that addressed the effect of a multimedia learning environment on the number concepts from one to ten on 20 preschool children, it was found that children were not successful in expressing number concepts (Cakiroglu & Taskin, 2016). In fact, the study concluded that although multimedia is increasing in society as a way to instruct and increase student performance, this is not a determining factor in improving student performance. Likewise, the study confirmed that gender was not an indicator for the development of number concept acquisition in children (Cakiroglu & Taskin, 2016).

Spoken and written number knowledge. The way numbers are spoken and written are quite different. When speaking, numbers are stated in the groups in which they belong. For example, for 225, we say “two hundred fifty-five”, identifying the base hundreds, tens, and ones. Numbers are typically stated as “one hundred, two hundred, three hundred, etc.” and not the way it can be written, 1 hundred, 2 hundred, 3 hundred, etc. When children begin counting the teen numbers, a new set of rules applies. The numbers are typically stated in reverse; for example, for 14, the four is said first in the number. Likewise, eleven and twelve, have no relation to one or two.

Children must master number knowledge words with representations before they can master verbal counting (Pinhas et al., 2014; Voustina, 2016). In a case study of a 6-year old child, Voustina (2016) identified the importance of social interaction in supporting number knowledge, and therefore, should be provided with structured activities that encourage knowledge and connections between counting, number structure, and calculation. Similarly, Pinhas et al. (2014) conducted a study of 150 preschool children on a variety of verbal, picture, and number word tasks to determine the neural processing of number words in the development of children learning the meanings of numbers and their placement in the number system, also known as the Approximate Number System (ANS). Results of this study indicated that preschool children correlated spoken number word comprehension with number words and their representations before learning and mastering verbal counting.

When writing numbers, understanding place value is essentially important. Numbers are designated into groups of ones, tens, hundreds, and so forth. Therefore, in 682, there are 6 hundreds, 8 tens, and 2 ones. Zero represents the place value where there is no number. For example, in 905, there are 9 hundreds and 5 ones. Eliminating the zero would change the number entirely; therefore, understanding that each number represents a designated group and position is important. Alvarado (2015) investigated written numerals of 45 preschool aged children on their ability to solve addition problems via counting. Children were assessed on their performance in counting, recognizing written numerals, and solving number conservation tasks. The results indicated that presenting students with written numerals facilitated their ability to perform addition problems (Alvarado, 2015).

Research identifies the important relationships between language and early numeracy skills (Purpura & Napoli, 2015; Purpura et al., 2015; Toll & Van Luit, 2014). In a study of 1,030 Dutch children, Toll and Van Luit (2014) investigated the development of basic language skills and low early numeracy skills. After a 2-year observation period, the researchers investigated the general language skills and early numeracy and found a strong relationship between the two skills for kindergarteners. It was concluded that math language is a key component in the early numeracy learning process. Purpura & Napoli (2015) concluded similar results in their study of 180 preschool children. This study aimed to assess print knowledge, vocabulary, information numeracy, and numeral knowledge from eight early numeracy measures; thus, resulting in a relationship between language and numeral knowledge through informal numeracy skills (Purpura & Napoli, 2015).

Informal numeracy skills. Purpura and Lonigan (2013) conducted a study of preschool children on a three-factor model of what is classified as informal numeracy skills: numbering, relations, and arithmetic operations. *Numbering* included verbal counting, counting forward and backwards, identification counting errors, structured counting, cardinality, counting a subset, subitizing, and estimation. *Relations* included ordinality, relative size, number comparison, set comparison, number order, sequencing, number or set reproduction, number identification, and numeration. *Arithmetic operations* included adding and subtracting with and without objects, initial equivalence, equivalent sets, and number combinations. These informal numeracy skill domains are imperative components for the identification and understanding of the basic number skills needed for the foundation of number mastery, such as counting, one-to-one correspondence, and quantity. Results indicated that the three informal numeracy domains were necessary for identifying mastery of early numeracy skills in preschool aged children. Most of the foundational mathematics learning takes place from ages 3-6 (Carpenter et al., 2017).

Early Childhood

Education of Early Childhood

Bassok et al. (2016) identify the number of resources (i.e. books and computers), resource accessibility, and parental investments for a child's early learning when enrolling their child in a childcare facility or early educational program; however, gaps still exist between parental knowledge and early childhood investment as suggested in the longitudinal study. It was accurately hypothesized that parents in 2010 believed their children needed to be more prepared and have more skills for kindergarten than parents from earlier years (Bassok et al., 2016). The study also identified specific academic skills needed for kindergarten readiness, such as counting to 20 and knowing letters to be among some of the largest skill increases. Bassok et al. (2016) reported using teacher data from the Kindergarten Teacher Survey on Student Readiness to identify that counting to 20 was a skill in which teachers felt children needed to be ready for kindergarten but continued to lack.

Early Childhood Data

Prior to 1990, a lack of data collection had been made on assessing academic achievement for children entering kindergarten. Generally, children were not assessed until they reached the third or fourth grades. Data on early learning experiences were incomplete, and although the National Assessment of Educational Progress (NAEP) attempted to regularly assess children's reading and academic achievement, assessment was largely provided for children after the age of 9 (West, 2017). In 1979, the National Longitudinal Study of Youth was conducted to assess the language, reading, math, social-emotional, behavior, and health status of a sample of children born to a group of teenage and young mothers over a course of seven years (West, 2017). The study was not a nationally representative study sample size; however, child assessment data was collected to determine growth. Unlike the assessment of school-aged children, this study was the only assessment utilized for collecting data among preschool aged children. Drawing conclusions about early care and educational experiences from nursery school enrollments, childcare participation rates, household surveys, and school and administrative record data, were the only formal data collections provided at the time. Moreover, there was an underestimated number of preschool aged children attending formal programs outside of the home for data accuracy (West, 2017). Limited data was collected to examine the impact of school and program characteristics on child outcomes.

Early Childhood Academic Achievement

Two articles examined the gaps in school readiness skills and achievement over a course of 12 years. Reardon and Portilla (2016) focused on the beginning school skills for children of the major ethnic groups (white, black, and Hispanic) who entered kindergarten between 1998 and 2010. The study focused on the traditional cognitive academic skills, such as reading and mathematics. Whereas, Cimpian et al. (2016) examined the gender gaps in mathematics for children entering kindergarten compared to several school years later. Gaps in school readiness existed early on when students entered kindergarten; however, some gaps have narrowed (Reardon & Portilla, 2016). Achievement gaps have narrowed for ethnic and socioeconomic disparity groups in mathematics; yet, there continues to be a 50% decline in learning for the same ethnicities.

Gender differences in learning approaches also exist. Male achievement gaps in mathematics are lower than their female counterparts with similar achievement and learning behaviors (Cimpian et al., 2016). Cimpian et al. (2016) found that gaps in mathematics for kindergarten girls were limited, but the gap widened over the first three to four years of school. This suggests that girls begin to have the necessary basic mathematical skills needed for academic performance in kindergarten but fail to develop a strong foundation for mathematical performance in later years. Larger gender gaps in mathematics are noticeable for children with stronger math skills; therefore, it is presumed that mathematic skills develop earlier than kindergarten (West, 2017).

Special Education and Early Intervention

Special education services are provided to all school-aged children if deemed necessary. Since the inception of the *Education for All Handicapped Children's Act* (P.L. 94-142) of 1975, the service of children with disabilities has increased (National Center for Education Statistics, 2018). According to the National Center for Education Statistics (2018), between 2011 and 2016, there was an increase from 6.4 million to 6.7 million in the number of students receiving special education services (p.1). There are thirteen disability categories that children are eligible to receive special education services (Office of Special Education and Rehabilitative Services, 2016). This study focused on preschool children ages 3-5; therefore, children identified for special servicing were eligible under the developmental delay category. This category is suitable for children ages 3 to 9 with varying unspecific disabilities who may be later eligible to receive schooling special education services under one of the 13 disability categories.

As special education has evolved over time, emphasis on early intervention services has been significantly recognized. These services are required for children who are eligible to receive special intervention or related services, mostly because of a child's functionality, birth history, general health, healthcare, and/or behavior. According to the *Individuals with Disabilities Education Act (IDEA)* of 1990, children with disabilities, ages 3-21, are to be provided a free and appropriate public education (FAPE), and early intervention services to infants and toddlers with disabilities. This introduction to special education identified the special education eligibility categories, the identification of a developmental delay, and the requirements of early intervention services provided by IDEA.

Interventions are needed for children with disabilities. This section will review the literature on domain-specific interventions (Davenport & Johnston, 2015; DuPaul et al., 2015; Hinton et al., 2016; Khomais, 2014) and computer-based interventions in mathematics (Mendizabal et al., 2015) as they have demonstrated improvements for preschool children. Research also questions the correlations between working memory and early numeracy skills in preschoolers (Kroesbergen et al., 2014; Kyttala et al., 2015; Passolunghi & Costa, 2016). Interventions for preschool children in these areas were reviewed for this study.

Special Education Disability Categories

Nine specific disability categories have been identified for service under Part B of the *Individuals with Disabilities Act* (IDEA) of 1990. Subsequent to multiple reauthorizations of the Act, the expansion of disability categories has increased to include thirteen categories: autism, deaf-blindness, emotional disturbance, hearing impairments, developmental delay, intellectual disabilities, multiple disabilities, orthopedic impairments, other health impairments, specific learning disabilities, speech or language impairments, traumatic brain injury, and visual impairments (Office of Special Education and Rehabilitative Services, 2016). According to the amended law in *IDEA* of 1997 (P.L. 105-17), states have been allowed to use the *developmental delay* category for children aged three through nine. Beyond age nine, children who continue to demonstrate significant academic and/or behavioral delays are classified according to one of the 13 categories notes above.

Moreover, the number of children with disabilities identified for public education services has increased since its inception in 1975. Much of the increase is attributed to an increase in students identified in having a specific learning disability. According to the National Center for Education Statistics (2018), there were a total of 6.7 million children identified as having a disability in the United States in 2014. The most prevalent eligibility category was specific learning disabilities, which comprises 34% of all students who receive services under IDEA, followed by speech and language impairments (20%) and other health impaired (14%). The number of children with disabilities is 13 percent of all public school students, with the majority of race/ethnicities being: American Indian/Alaska Native (17%), Black (16%), White (14%) two or more races (13%), Hispanic (12%), and Pacific Islander (12%) (National Center for Education Statistics, 2018). Unfortunately, the number of children, ages three through five, with disabilities or delays in each categorical group is undefined due to inaccuracy and deficiencies in reporting.

Developmental delays. With approximately 514,193 males and 221,603 females ages 3 through 5 in 2014 receiving special education services, the statistics do not classify the significant categorical groups because it has been noted that many children under the school-age of five are classified under one umbrella: a child with a developmental delay (Digest of Education Statistics, 2016). States often used the terminology "developmental delay" as an option for children ages 3 through 9 for differences in reporting practices (Office of Special Education and Rehabilitative Services, 2016). A developmental delay is a condition in which a child is less developed mentally or physically than for his/her normal age peers. For the purpose of this study, children in preschool are typically not identified with a specific disability; however, they can be identified as having a developmental delay and may later qualify for special education services under of the 13 disability categories.

Developmental disabilities. Developmental disabilities are severe chronic conditions due to mental and/or physical impairments and they last throughout a person's lifetime (Boyle et al., 2011). Research from the Centers for Disease Control and Prevention (CDC) and the Health Resources and Services Administration (HRSA) have identified the prevalence of developmental disabilities in the United States from 1997-2008. Based on the findings, 1 in 6 children, ages 3 to 9 in the United States have or have had a developmental disability, increasing to 15% over 12 years (Boyle et al., 2011). As a predominantly male category, specific developmental disability categories, such as autism and attention deficit hyperactivity disorder, have also increased for children under the age of 9 (Boyle et al., 2011).

Early Intervention Servicing

According to *IDEA*, infants and toddlers identified for early intervention may receive services and continue to receive related services subsequent to three years old. Eligibility requires children between 0 to 2 years of age are placed on an individualized family service plan (IFSP). The average age of a child identified for an IFSP for eligibility services was 17.1 months (Office of Special Education and Rehabilitative Services, 2016). Approximately 64% of children eligible for early intervention services were likely to begin services after 21 months of age. Children under five not classified as having a developmental delay but required services at 12 months of age or less, needed services because of prenatal or perinatal abnormalities or speech and/or communication deficits (Office of Special Education and Rehabilitative Services, 2016). Children were namely referred for early intervention services because of or lack thereof the following: functionality, birth history, general health and health care, and/or behavior.

At three years of age, children are reevaluated to determine if *IDEA* Part B services are necessary. According to the Office of Special Education and Rehabilitative Services (2016), 61.2% of children in 2014 evaluated at age 3 were exited from early intervention services, 17.9% were eligible to receive special services but eligibility was undetermined, 4.9% were eligible with the same eligibility category, and 16% were not eligible. In 2014, there were 350,581 infants and toddlers and 753,697 children ages 3 through 5 who were served under *IDEA* (Office of Special Education and Rehabilitative Services, 2016). The number of children receiving services has increased seven percent since 2005. The most prevalent disability categories for children ages 3 through 5 include: speech and language impairments (43.7%), developmental delay (37%), and autism (8.9%) (Office of Special Education and Rehabilitative Services, 2016).

In 2014, there were a total of 65.8 percent of children ages 3 through 5 in regular early childhood programs, 23% in separate classes, and 4.8% in other educational environments (Office of Special Education and Rehabilitative Services, 2016). Although the *IDEA* mandates the identification and the provision of services to children with disabilities ages 3-21, the number of children identified for early intervention services between the ages of 3 through 5 is significantly low for a variety of reasons. It is suggested that children ages 3 through 5 are excluded from special education identification for one of the following reasons: family structure and/or socioeconomic status, resources availability, and parental education (Office of Special Education and Rehabilitative Services, 2016). Other concerns, excluding child intervention services, include: children do not attend preschool or daycare and have not been identified by an education professional; the disability has not been identified by a doctor; the disability has not presented itself because of age; parental denial or refusal of evaluation and/or identification of a disability; and/or the disability has not affected academic performance.

Interventions

As children struggle in specific academic areas, interventions are needed to provide support and reteach skills for further practice and understanding. Research in many areas of learning have focused on the need to include evidence-based core or supplemental interventions. More specifically, the research in mathematical interventions continue to evolve; these interventions are either content focused or computer based (Mononen, Aunio, Kopenen, & Aro, 2014). As a result, there are promising effects of interventions, such as explicit instruction, computer-assisted instruction, game playing, and using concrete-representational-abstract in improving the early numeracy skills of children with and without math disabilities (Mononen et al., 2014). This section of the literature review will identify domain-specific interventions, computer-based interventions, and how working memory correlates to early numeracy skills in preschool children.

Domain-specific interventions. Domain-specific interventions are interventions that are focused on a specific content area, such as mathematics. Consequently, early interventions focused on domain-specific interventions in preschool demonstrated improvements in mathematics for children (Davenport & Johnston, 2015; DuPaul et al., 2015; Hinton et al., 2016; Khomais, 2014; Kroesbergen et al., 2014; Kyttala et al., 2015; Passolunghi & Costa, 2016). As explicit instruction is a form of domain-specific instruction, Hinton et al. (2016) examined the effects of supplemental explicit instruction on the counting performance of four preschool students who were identified as at-risk for mathematics failure in the future. The results of this study found that all students reached the benchmark across the counting skills. Similarly, from a sample of 135 preschool children with attention deficit hyperactivity disorder, DuPaul et al. (2015) investigated how child, family, and treatment variables predicted treatment outcomes for reading and mathematics achievement and behavior over a 24-month period. The results indicated overall growth in math performance based on age, cognitive ability, gender, and lower family support. Davenport and Johnston (2015) also conducted a study that examined the effectiveness of an intervention strategy (prompting and contingency management) but was geared toward creating opportunities, prompting, providing consequences, and fading for three preschool children with disabilities in a single-subject design. The results of the study indicated that the intervention strategy was effective, and the researchers discussed the importance of using this strategy in inclusive preschool classrooms to support domain-specific interventions (Davenport & Johnston, 2015).

Additionally, Mononen et al. (2014) conducted a review of various studies on the impact of early numeracy interventions for 4 to 7-year old children at risk for mathematics difficulties. The interventions included explicit and guided instruction, activities involving concrete-representational-abstract, small-group instruction, and computer-based interventions. All forms of interventions demonstrated the effectiveness of the studies. Moreover, approximately 30% of the supplemental interventions applied concrete-representational-abstract approaches (Mononen et al., 2012).

Computer-based interventions. Specific computer-based training programs have also been used as an intervention to improve early mathematical skills (Mendizabal et al. 2015; Clements & Sarama, 2009). In a study of 128 preschool children that included 30 sessions of a remedial intervention of playing with numbers, early mathematic competency of children at-risk of having a learning disability were assessed. Significant differences between groups existed as children who received the intervention improved more on the Early Numeracy Test than students who did not receive the remedial intervention. Another study evaluated the efficacy of a preschool mathematics program called Building Blocks, on 68 preschool children (Clements & Sarama, 2009). In this pre- and post-test study, students who used the program performed significantly better than those who did not. Likewise, Papadakis et al. (2017) conducted a study that investigated the influence of computers and tablets in the development of mathematical competence on 256 preschool children in Greece during a 14-week study. The study concluded that teaching with a tablet and teaching from the computer contributed significantly to the development of children's mathematical ability. Overall, using computer-based interventions demonstrated positive improvements in developing mathematical concepts for preschool children.

Working memory. Research questions the correlations between working memory and early numeracy skills in preschool children (Kroesbergen et al., 2014; Kyttala et al., 2015; Passolunghi & Costa, 2016). Kroesbergen et al. (2014) conducted a four-week study on 51 preschool children with either a working memory training or no training at all. Pre- and post-test analyses concluded that children who participated in a working memory intervention improved their early numeracy skills. Passolunghi and Costa (2016) had similar results in their study that aimed to verify and compare the effects of two types of early numerical skill interventions (early numeracy skill intervention and working memory intervention) on 48 preschool children for five weeks. The results indicated that both interventions improved early numeracy abilities in preschool children (Passolunghi & Costa, 2016). The correlation between working memory and counting skills was also questioned in Kyttala et al. (2015). In a study of 61 preschoolers, the effects of two training conditions (counting training and simultaneous training of working memory and counting) were investigated. Kyttala et al. (2015) found that domain-specific training, such as counting, in mathematical skills is more effective in improving early numeracy skills than combining working memory and counting together. These studies demonstrated the correlation between early intervention and counting skills as precursors for early prevention of learning difficulties and disabilities during preschool years.

Influence of Teachers in Mathematics

Many factors can influence a child's attitude towards mathematics. These factors include but are not limited to the teacher's attitude towards the environment, beliefs and practices, and a teacher's preparedness for instruction (Bulder & Omeroglu, 2018; Madu, 2016; Stipek, Giwin, & Macgyvers, 2001; Takunyaci & Takunyaci, 2014). As teaching is a predominantly female profession, and research proves that girls are less interested and tend to do more poorly in mathematics than boys, it is important to understand how teachers influence the mathematical environment (Mutjaba & Reiss, 2016; Tichenor et al., 2016).

Specifically, in a preschool classroom, teachers have a great impact on how children learn and perform (Bulder & Omeroglu, 2018). In a recent study of 26 preschool teachers, it was determined that preschool teachers' attitudes towards problems in the environment was not as high as their awareness of the problems (Bulder & Omeroglu, 2018). It is important to understand that teachers have a high awareness of issues within their environment and do not easily overlook identifying problems and solutions. This is important as children's attitudes towards their environment and level of knowledge are high influences in the behavior of the environment.

Research also reports a positive correlation between teacher beliefs and practices related to mathematics (Stipek et al., 2001; Takunyaci & Takunyaci, 2014). In a study of 21 fourth-sixth grade teachers at the beginning and end of the school year, teachers self-reported that there were consisted associations between their beliefs about mathematics and teacher instructional practices. It was also reported that teachers' self-confidence in mathematics was also significantly associated with a student's self-confidence as learners. Similarly, in a study of 95 preschool teachers, Takunyaci and Takunyaci (2014) investigated preschool teacher's efficacy beliefs regarding mathematics teaching using the Mathematics Teaching Efficacy Belief Instrument in Turkey. The results of the study indicated that teachers have low efficacy beliefs on teaching mathematics, more specifically, teachers without strong beliefs generally teach mathematics ineffectively. There was also a relationship between positive teachers' efficacy beliefs and over 13 years of experience int teaching preschool.

Similarly, a lack of teacher preparedness for instruction is related to ineffective teaching. Madu (2016) conducted a study of students at 211 secondary school in Kano State to investigate mathematics teacher's preparedness and effectiveness of students' point of view. The results of a questionnaire indicated that mathematics teachers' ineffectiveness is due to the teachers' inadequate preparation for lessons and instruction. Therefore, the study's findings suggested that teachers should equip themselves with instructional strategies that will improve their preparation and effectiveness.

Mathematical Skill Development of Children

As children develop through the stages of mathematical learning, also referred to as developmental/learning paths, assessing and interpreting the mathematical development in the classroom lead to individualized learning experiences (Dunphy et al. 2014). Based on the empirical study by Papadakis et al. (2017) in which 231 kindergarten children were either taught Realistic Mathematics or basic pedagogical principles of curriculum for kindergarten children, the Test of Early Mathematics Ability (TEMA-3) indicated that the teaching techniques of Realistic Mathematics contributed significantly to the development of mathematical concepts of younger children.

Influential Factors for Mathematical Growth in Children

As children develop mathematical skills in earlier years, there is a need to understand the factors that influence mathematical growth. Such factors include: the learning environment, mathematical tools, classroom discourse, mathematical tasks, and assessment (Varol & Farran, 2006). Without these factors, the improvement and success of mathematical achievement in children would be limited.

Learning environment. As children enter the classroom environment, it is important for teachers not to limit access to materials that challenge a child's growth in mathematics (Dunphy et al., 2014; Varol & Farran, 2006). The quality of the classroom environment is one factor that influences the improvement of educational outcomes for children (Wang, Haertel, & Walberg, 1993; Waxman & Huang, 1997). In the classroom environment, there are two types of learning environment characteristics: external and internal characteristics (Chapin & Eastman, 1996). As children absorb and learn in a classroom, the furniture arrangement and materials utilized play a significant role in student behavior and academic success (Varol & Farran, 2006). After analyzing a set of kindergarten classrooms and the environmental characteristics on student development, it was found that adequate classroom resources improve socialization, behavioral health, and parental involvement (Bennett, Elliott, & Peters, 2005). Although these external environmental characteristics are beneficial for a child, they do not support mathematical outlooks. The interaction of the classroom teacher, and his/her beliefs, values, attitudes, and knowledge of mathematics, are an integral component to transforming the learning environment and providing challenging and successful opportunities for mathematical ability to develop (Emenaker, 1996; Goddard, Hoy, Hoy, & Woolfolk-Hoy, 2000; Rimm-Kaufman & Sawyer, 2004; Varol & Farran, 2006). Elementary teacher's beliefs about mathematics effect the teaching of mathematics (Emenaker, 1996). Studies support a link between teacher beliefs of mathematics and student achievement (Goddard et al., 2000; Rimm-Kaufman & Sawyer, 2004); therefore, there is a need for elementary teachers to hold positive beliefs about mathematics for effective teaching to take place.

Mathematical tools. The emphasis on using mathematical objects for teaching mathematical skills has been an instrumental component and factor in early childhood education for many years. Many historical theorists in early childhood education have highlighted the importance of objects in foundational mathematical skills, such as Montessori, Froebel, and Dewey (Wolfe, 2002). Researchers today agree with these theorists that mathematical instruction is most effective when concrete materials are utilized (Ball, 1992; Furner & Worrell, 2017; Laski et al., 2015; Steedly, Dragoo, Arafeh, & Luke, 2008; Thompson, 1994; Uttal et al.,1997). The use of concrete and hands-on materials is an exciting and engaging way of learning for children. Objects are tangible and can help children develop images in their minds for later reference. When learning numeration skills, children use their body as an easily accessible tool, such as their fingers for counting (Hunting, 2013). In addition to the human body, children can use objects and manipulatives, such as counting bears and base-ten blocks for counting single numbers and learning other mathematical components.

Although the use of manipulatives and other hands-on objects has been an ideal component in the development of mathematical skills, research has not supported consistency in using these mathematical tools for instruction over traditional instructional methods of paper and pencil and rote memorization (Varol & Farran, 2006). Children may have a difficult time understanding the relationship between manipulatives and mathematical symbols and numbers. Resnick and Omanson (1987) demonstrated how children have difficulty in solving written problems when solely introduced to blocks. These researchers also concluded that based on the results, children have a weak understanding of mathematical skills and their understanding can be easily lost. The ability to master computational and mathematical skills is difficult for children learning new mathematical concepts; hence, the difficulty for children to recall mathematical skills learned from one day to the next (Varol & Farran, 2006). These mathematical objects may well be considered a tool for children to use for playing with rather than a direct understanding of mathematical skill development (Ball, 1992). Therefore, it is important to understand that similar to the learning environment, mathematical manipulatives are not solely responsible for mathematical success and achievement in children.

Classroom discourse. As children learn mathematics, it is critical for teachers to understand their learning style and thinking skills. As children process the information, inquire and strategize, effective classroom discussions are prompted. In traditional classroom environments, classroom discourse occurs through spoken language (Varol & Farran, 2006). Thus, teachers and students have diverse roles in the of discourse process. A teacher's role is to present new material (Sherin, 2002), and the student's role is to listen, observe, and evaluate the newly absorbed knowledge based on prior knowledge and experiences (Bruner, 1996). During these discussions, children share their ideas and solutions to mathematical problem solving (Sherin, 2002). Moreover, classroom discourse influences problem solving and reasoning skills (Lappan & Schram, 1989). A teacher's goal is to provide effective learning. Effective learning takes place when a teacher creates an environment where children can freely discuss their ideas and activity participate. Overall, it is important for teachers to connect the mathematical tools and materials with effective tasks, in order to build effective classroom discourse (Varol & Farran, 2006).

Mathematical tasks. Through the utilization of developmentally appropriate mathematical tasks, effective classroom discourse is ensured (Varol & Farran, 2006). Developing and implementing meaningful tasks increases student motivation and their ability to connect concepts in mathematics to real-world situations (Middlleton & Spanias, 1999; Williams-Pierce, 2011). Providing children with problems that encourage active engagement and developing their own strategies is ideal for children developing skills in mathematics.

Assessment. With a variety of assessment techniques and tools, formative assessment is regarded as the most effective for creating an accurate image of a child's mathematical learning (Carr & Lee, 2012; Dunphy et al., 2014). Teachers spend approximately one-third of the time on assessment activities in an attempt to identify student skills, improvement, and feedback (Pitcher, Goldfinch, & Beevers, 2002). Assessment is an effective tool needed for guiding instruction, monitoring student improvement and growth, and assessing learning (Anderson & Palm, 2017; Buhaglar & Murphy, 2008). As children progress in mathematics, assessments should be utilized to identify what students are capable of and not capable of doing (National Council of Teachers of Mathematics, 2003). Especially for children in grades K-2, assessment should be utilized for evaluating student progress. In 2003, the National Council of Teacher of Mathematics (NCTM) identified five mathematics assessment standards that are used for monitoring student progress, instructional decisions, and evaluating overall student achievement in mathematics. These standards indicate learning, equity, openness, inferences, and coherence as it pertains to assessment (NCTM, 2003). Assessments should be geared towards: what children can do; an opportunity for all children to demonstrate their abilities; an open process in which all stakeholders are informed about the nature of the assessments so all who are involved in the student's learning have an opportunity to understand the criteria for mathematical assessments; the promotion of analyzing student mathematical learning; and all assessments should be aligned to curriculum and instruction (NCTM, 2003). The NCTM standards serve as a guideline for assessment criteria and to improve classroom quality.

Gender. There are varying gender differences that exist with learning; however, one of the most commonly identified differences is the achievement gap in mathematics between males and females (Cimpian et al., 2016; Jelas & Dahan, 2010; Khaleel, 2017; Tichenor et al., 2016; West, 2017). Males are identified as having stronger foundational skills in math than females (Cimpian et al., 2016; West, 2017). Gaps in mathematical achievement continued to widen over the first three to four years of school. It is presumed, that based on these gaps and the noticeability for children with stronger math skills, mathematical skills are developed earlier than kindergarten (West, 2017).

As other factors influence growth in mathematics, one specific and significant influence suggested on the performance in mathematics is gender. Girls are more likely to do better academically than boys (Jelas & Dahan, 2010; Khaleel, 2017). However, research suggests that girls are less interested in mathematics and therefore, are outperformed by boys (Mutjaba & Reiss, 2016; Tichenor et al., 2016). A small-scale study conducted from teacher interviews based on the performance of boys and girls in mathematics revealed that girls are less interested in mathematics than boys and should be encouraged to do mathematics to improve their career choices later in life. Expectations pose a great response to mathematics learning. Children who have higher mathematics aspirations perform better in mathematics and have higher motivation than students who do not (Mutjaba & Reiss, 2016). However, regardless of aspirations and motivation, girls are reported to less likely perform better than boys in mathematics (Mutjaba & Reiss, 2016; Tichenor et al., 2016).

Mathematical Skill Development of Children with Disabilities

Research indicates that many children, whether faced with a disability, from low socio-economic status, or due to other circumstances, struggle with mathematical concepts (*e.g.,* Bashash et al., 2003; Jordan & Levine, 2009; Peterson & McNeil, 2013; Purpura & Lonigan, 2013). Studies support the use of hands-on materials when learning mathematical concepts particularly for students with learning disabilities (Bennett & Rule, 2005). In Bennett and Rule's (2005) study, 27 middle school students with disabilities used skittles, base ten blocks, colored counters, or numeral cards, or computerized drill and paper/pencil work to solve ten division problems. There was a significant difference between the average gain scores of the two groups in favor of the experimental group (17.1%) versus the control group (4.5%). The study findings indicated that students using manipulatives scored better on all testing sections than students using the traditional paper and pencil.

Standard instruction has been shown to be ineffective for students with mathematical learning disabilities (Lewis, 2016; Mazzocco et al, 2013). Lewis (2016) included a case study of two students with mathematical learning disabilities who had one-on-one tutoring sessions. Analyses revealed that these students had difficulty understanding mathematical problems when presented with traditional instructional methods. Their understandings directly contributed to their tenacious struggles in mathematics (Lewis, 2016). The analysis revealed that students with mathematical learning disabilities understand mathematical representations in atypical ways because of their learning difficulties.

Clements and Sarama (2009) stated that children with disabilities appeared to have confidently recited counting; however, this simply reflected observational learning as responses were memorized and not mastered. Results from Talbot, Ahmad, and Ghazali (2013) indicated that students with mild intellectual disabilities had difficulty with number counting related to abstract and irrelevant order principles. Abstract and irrelevant principles are less concrete and more abstract as children are required to use critical thinking skills. Results were most favorable on the one-to-one principle (70%), stable order principle (67.5%), and cardinality principle (67.5%). In addition, Talbot et al. (2013) found no differences between genders.

Further research is needed to identify the levels of achievement in numeracy for students with varying intellectual abilities and developmental delays. In a systematic study investigating basic counting and number skills, strategies used for counting, and number tasks by students with moderate intellectual disabilities ages 7-18, Bashash et al. (2003) found that the tasks presented to the students (sequence, counting, cardinal, and symbol) represented early stages in the development of counting and numbers. Children with intellectual disabilities are able to achieve basic counting skills and principles; however, deficits continue to exist (Bashash et al., 2003), particularly in one-to-one correspondence. Bashash et al. (2003) also concluded that preschool children are less successful in determining whether a number was presented in a sequence; therefore, it is important to identify if children with varying exceptionalities and typically developing children also struggle to successfully determine whether a number is in sequence or not. Children who struggle at the early stages of learning with mathematical concepts are highly likely to continue to have difficulties in mathematics later in school (Jordan, & Levine, 2009). These difficulties could be later identified as a math disability known as dyscalculia.

Dyscalculia. Dyscalculia is a learning disability that impedes the learning of numeracy skills (Price, 2013). This disorder affects a child's ability to learn arithmetic skills in school. According to research presented by Price (2013), dyscalculia affects approximately 3-6% of children in the United States. Children who demonstrate a behavioral deficit in one or more mathematical skills, can be identified for a learning disability, such as dyscalculia. The lack of calculation strategies and poor mathematical foundations are features of dyscalculia (Price, 2013). Deficits in basic number processing, arithmetic, and non-numerical problems are experienced in children with dyscalculia.

Children present mathematical skill deficits in one of two forms. Math deficits may be caused by an impaired ability to acquire mathematical skills or external factors (Price, 2013). Children who have an impaired ability to acquire mathematical skills are classified as having primary dyscalculia. Children who are posed with hindering external factors are considered as having secondary dyscalculia. Some disabilities, such as autism, could be comorbid with an intellectual disability related to mathematical disabilities (King et al., 2016).

King et al. (2016) conducted a meta-analysis to identify math interventions that examined 28 articles involving children and adolescents with Autism Spectrum Disorder and comorbidity with an intellectual disability. Although the study identified positive findings for interventions for computational (39%) and functional skills (39%), none of the interventions were considered evidence-based practices (King et al., 2016). Early numeracy and problem solving were targeted in 7% of the cases involved. Therefore, further research and empirical studies related to mathematics and students with disabilities must be conducted to identify effective and appropriate evidence-based practices.

Other forms of manipulatives or assistive technology have demonstrated effectiveness for specific disabilities. Hudson et al. (2015) conducted a multiple probe design on three elementary-aged students with multiple disabilities that evaluated the effects of an early numeracy systematic instructional package intervention using a single-switch output device and other manipulatives to determine early numeracy skill acquisition. Twelve early numeracy skills were fixed into the lessons and from baseline to intervention. Results showed that the manipulative interventions had a positive and immediate effect on all participants' acquisition of early numeracy skills.

Mathematics Curriculum

As children develop mathematical reasoning, skills, and understanding, they need to be provided a high quality and challenging mathematics education at an early age (National Council of Teachers of Mathematics [NCTM], 2003). Early experience of mathematics influences later mathematical performance (NCTM, 2003). Research supports the need to improve mathematical curriculum that challenge students (Foreman & Gubbins, 2015; Martin et al., 2017). As many students are required to take remedial developmental mathematics courses in high school and college to gain a stronger foundation of skills and knowledge, many continue to struggle to complete the coursework and are unable to graduate. This lack of foundation begins at an early age and continues throughout a child's adulthood. Martin et al. (2017) analyzed a large set of data on developmental mathematics assessments and curriculum and determined the need to improve the developmental mathematics curricula. Foreman and Gubbins (2015) assessed 1,413 third grade students on pre- and post-mathematics tests to determine if challenging mathematics content reinforced student performances and overall teacher nominations for the gifted and talented identification system. Results confirmed that students in the treatment group who received challenging content outperformed their peers in the mathematics post-test and were perceived by their teachers to have qualities beyond their peers.

In addition to providing high quality and challenging curriculum, research supports the need for children to participant in interventions focused on working memory skills to absorb the necessary knowledge for counting. Kyttala et al. (2015) proved that cognitive abilities play an important role in early numeracy. Based on her study, the effects of two training conditions (counting training and training of working memory and counting) were investigated to determine the effectiveness of these training on 61 preschool children. The results indicated that preschool-aged children do not benefit from a short period group training of working memory skills; in fact, training of working memory is not effective (Kyttala et al., 2015). The study concluded that domain-specific training in specific mathematical skills was more effective in improving early numeracy performance than focusing on working memory and counting training.

Traditional Mathematics

Traditional mathematics curriculum is used across the world in a variety of forms. Some of the most common mathematical curriculum includes the following: visual representations, concrete-representational-abstract, digital, and the use of manipulatives.

Visual. Integrating visual representations in the mathematics curriculum offers a different kind of perspective to understanding the foundation (La Haye & Naested, 2014). La Haye and Naested (2014) discussed the importance of connecting visual art into the mathematics curriculum because traditional teaching fails to address diverse learning styles of students.

Concrete-representational-abstract. The concrete-representational-abstract (CRA) framework of instruction is also promising for students with disabilities (Agrawal & Morin, 2016). This evidence-based approach provides intervention for mathematics instruction, specifically for students with learning disabilities. The three-stage instructional sequence (concrete, representation, and abstract) begins with a teacher modeling the concept and material, followed by the teacher transforming the concrete model into a drawing, and completed by the teacher modeling the concept in numbers and symbols (Flores, 2010). Agrawal and Morin (2016) recognized the use of concrete manipulatives to support instruction for students with intellectual disabilities and connected the necessary conceptual and procedural knowledge for mathematical concepts. Consequently, Soydan (2015) indicated that six-year-old children acquire a higher degree of operational skills when presented with hands-on educational materials. This instructional approach is aligned with the Montessori and Piaget theories of learning. However, CRA focuses on the teacher instructing versus the child's learning abilities. Therefore, it is important to identify the components of the Montessori Method that can be effective for a preschool child's learning and development of early numeracy skills as both the CRA and Montessori approaches have varying similarities.

Mathematics manipulatives. Manipulatives in mathematics are important components for classroom instruction. Using hands-on manipulatives have been supported across multiple research studies. More specifically, research supports the use of concrete objects in a child's counting performance (Peterson & McNeil, 2013). Peterson and McNeil (2013) suggested that when children utilized known objects in counting, their performance could be hindered ($p = .04$); therefore, it is important to utilize objects in which a child lacks familiarity.

Additionally, research identifies the importance of mathematical manipulatives for children ages 3 through 6 (Carbonneau et al., 2013). Statistically significant results in 55 studies indicated the need to integrate concrete manipulatives in daily mathematical instruction ($p < .001$), the development of addition skills and manipulatives ($p = .002$), and learning outcomes and problem solving ($p = .01$) (Carbonneau et al., 2013). These research findings demonstrated the need to identify evidence-based interventions using hands-on manipulatives for instructing young children (Carbonneau et al., 2013; Peterson & McNeil, 2013).

Digital manipulatives. Manipulatives have also been extended to the virtual realm. During the 20^{th} century, digital technologies were introduced into mathematics curriculum. As of today, the use of digital technologies is widely utilized to teach and learn mathematics and have proven to be successful (Lagrange & Kynigos, 2014). Early childhood education teachers are using both concrete and virtual tools to support early learning skills. Many studies have identified the positive impacts of using virtual manipulatives through computers for mathematical learning (e.g., Moyer-Packenham & Westenskow, 2013). In a study by Mattoon, Bates, Shifflet, Latham, & Ennis (2015), four and five-year-old children were randomly assigned to a traditional manipulative group and a virtual manipulative group. Both groups had an increase in computational skills when presented with traditional manipulatives and digital manipulatives (Mattoon et al., 2015). Therefore, the use of any manipulative can support student understanding and knowledge of symbols and mathematical concepts (Rosli et al., 2015). Similarly, Desoete, Praet, Van de Velde, Craene, and Hantson (2016) claimed there was evidence to support the use of virtual manipulatives to improve preschool children's early mathematics skills. Utilizing mathematical manipulatives rather than traditional methods was also recommended. Likewise, Soydan (2015) concluded that there is a difference in acquisition and operations skills of preschool children when presented with educational toys and smartboard techniques versus traditional methods.

Based on the use of manipulatives for classroom instruction and student learning, research supports that children perform better when given the opportunity to utilize hands-on tools for learning (Bouck et al., 2013). Bouck et al. (2013) conducted a study on sixth and seventh grade students with and without disabilities to identify if using a calculator improved their performance on solving mathematics assessment problems. Results of the study indicated that although the amount of time spent on the test was reduced for all students, students who self-reported using a calculator answered questions correctly compared to those who did not use a calculator.

The Montessori Method of Mathematics

The techniques of the Montessori Method follow the child's natural physiological and psychical development and is divided into three main parts: (1) motor education, (2) sensory education, and (3) language (Montessori, 1964). Motor education is based on the management and care of the environment; while, sensory education and language are supported through the use of didactic materials. Montessori developed and utilized sensory and didactic materials in a specific order to teach young children (Montessori, 1964). These materials, whether differing in size, unit of measure, thickness, or color, were compared to one another as they are in the series of numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Didactic materials are important for children to understand clearly the idea of a *number*. Montessori (1964) identified that numbers existed as an object and can be combined together to form other numbers. The use of didactic materials in teaching numbers was designed for children to touch each object in the direction in which they write and to name each object at the same time. As zero is nothing, this concept is taught separately to children in a Montessori setting so that children understand this comes before the number one. Teaching the concept of zero requires the use of a didactic tool, called the Spindle Box, in which a child places the correct number of spindle sticks into the corresponding number in a wooden box. Numbers 0 through 9 are represented. In the position of zero, nothing will be placed there; thus, demonstrating the number zero.

Research supports an overall positive impact of the Montessori Method on student mathematical learning and outcomes (Kayili & Ari, 2011; Kayili, 2018; Lillard, 2012; Lillard & Else-Quest, 2006); however, some studies have proven to be inconsistent (Brown & Lewis, 2017; Lopata et al., 2005; Manner, 2006). More specifically, literature supports the use of the Montessori Method in learning geometrical shapes (Ongoren &

Turcan, 2009), the development and understanding of place value for acquiring numeracy skills (Bennett & Rule, 2005; Mix, Smith, Stockton, Cheng, & Barterian, 2017; Reed, 2000; Young-Loveridge, 2001), sequencing and number knowledge (Chisnall & Maher, 2007), building logical thinking skills (Haq & Alfilfili, 2015), and the use of manipulatives (Laski et al., 2015; Varol & Farran, 2006).

Overall mathematics achievement. Despite the documented positive impact of the Montessori Method, overall mathematics achievement has proven inconsistent results for students receiving a Montessori education (Brown & Lewis, 2017; Kayili & Ari, 2011; Kayili, 2018; Lillard, 2012; Lillard & Else-Quest, 2006; Lopata et al., 2005; Manner, 2006). Kayili and Ari (2011) investigated the effects of the Montessori Method on 50 preschool children's readiness to primary education. Results of the study concluded that the Montessori Method had a positive impact on preschool children's readiness to primary school.

In an experimental study, Kayili (2018) evaluated a sample of 60 children from a preschool in the United Kingdom to investigate the effect of the Montessori Method on the cognitive temp (the way a child behaves when he/she has to choose among a variety of alternatives) of preschool children. Data from the study was collected and results indicated that subsequent to six weeks of treatment, children who used the Montessori Method decreased a larger number of errors on mathematics problems when compared to the numbers of errors decreased for children in the treatment group.

Despite these positive findings, the overall mathematical achievement in older children was not significantly enhanced by the use of Montessori curriculum and materials in studies conducted by Brown and Lewis (2017), Lopata et al. (2005), and

Manner (2006). Brown and Lewis (2017) recently conducted a quasi-experimental study that compared reading and mathematics achievement for African American third grade students in Montessori and magnet schools in North Carolina. The results indicated no significant difference of mathematics scores for African American students. In a study comparing the academic achievement of fourth through eighth grade students in Montessori and traditional education programs, findings rejected the hypothesis that enrollment in a Montessori school was associated with higher academic achievement for older students.

Geometrical shapes. The Montessori Method has demonstrated effectiveness in learning geometrical shapes for non-disabled children ages 4-5 (Ongoren & Turcan, 2009). Ongoren and Turcan (2009) explored the effects of the Montessori Method on the acquisition of knowledge about geometric shapes in a six-week pre-test/post-test study of 40 participants. There were significant differences between pre- and post-test scores on the Concept of Geometric Shapes assessment for the experimental group, the control group, and for the overall use of the Montessori Method ($p < .001$). The Montessori Method demonstrated effectiveness for preschool children learning geometric shapes in these studies when compared to non-Montessori methods.

Place value. Further research supported the use of Montessori Methods and its positive impact in mathematics instruction, specifically in place-value understanding (Bennett & Rule, 2005; Mix et al., 2017; Reed, 2010). Bennett and Rule (2005) identified the positive impact of using color-coding Montessori materials to highlight place value concepts and long division for middle school students with learning disabilities in mathematics. Results indicated that students with mathematics learning disabilities performed significantly better using Montessori manipulatives than students who did not use Montessori materials (Bennett & Rule, 2005). Reed (2000) compared the place-value understanding of 93 first through third grade Montessori and non-Montessori classes and found statistically significant differences in favor of Montessori students on conceptual tasks. Similarly, in an experimental study that investigated whether concrete models supported place value learning for 7-year-old children, Mix et al. (2017) concluded that Montessori students demonstrated better understanding of base-ten structures than their mainstream peers at traditional elementary schools. In the Montessori Method's place value development, most children observed and experienced the Montessori bead materials, which was intended to develop conceptual numeracy development.

Sequencing. The Montessori Method in mathematics has a strong focus on sequencing and order. Chisnall and Maher (2007) sampled 62 four- and five-year-old children from 34 Montessori preschools and 28 traditional preschools. Differences between Montessori and traditional preschool performances were identified. In the backward number word sequence task, children in Montessori schools performed significantly higher ($p < .01$) than those in traditional schools. As backward number word sequence is the foundation for subtraction, it is important to continue research in this task area. However, it is unsure whether the Montessori bead material, other concrete materials, or Montessori facilitators are assisting with developing the skills for this strategy (Chisnall, & Maher, 2007). This study further concluded that preschool children in the Montessori settings outperformed students in traditional settings on place value.

Logical thinking. Critical thinking is a difficult skill to develop no matter tha mathematical curriculum presented. Recently, Haq and Alfilfili (2015) conducted a study of 58 non-disabled kindergarten children in Montessori and traditional school environments that compared the development of logical thinking skills. Although no significant differences existed between genders, the findings showed significant differences in logical thinking scores in favor of students using the Montessori curriculum ($p < .01$).

Manipulatives. In a Montessori classroom, manipulatives are needed for child discovery and learning. Laski et al. (2015) identified four principles for maximizing the effectiveness of manipulatives: (1) use a manipulative consistently and for a long period of time; (2) begin with transparent concrete representations and move to more abstract representations; (3) avoid everyday objects of distracting materials; and (4) explain the relation between the manipulative and the mathematical concept. Overall, it is important to provide high quality mathematics instruction and intervention for all children with and without disabilities to prepare each child for the future. Varol and Farran (2006) denoted a child's classroom environment, mathematical tools that foster and facilitate learning, effective classroom discourse, and the use of mathematical tasks for increased motivation on particular concepts and tasks. Research supported the use of manipulatives for teaching early numeracy skills for preschool students with and without developmental delays.

Montessori mathematics curriculum. Beyond the actual structure of the environment and the role of the teacher in a Montessori environment, there is the presentation of the activities and the overall curriculum. Between the ages of 0-3, children in a Montessori setting have been exposed to a variety of sensorial materials (Pickering, 1992). These materials are required for mathematics development.

Before understanding and being exposed to abstract materials, a child is exposed to the discrimination of the sensorial curriculum (Pickering, 1992). The sensorial curriculum is a sequential order of a set of wooden materials used to train and improve children's senses (Montessori, 1967). The sensorial is different from sensory play as it is controlled, structured, accessible at all times, and the materials never change. The base of the Montessori mathematics curriculum includes the study of numbers, linear counting, skip counting, and place value. A carefully designed and unique lesson has been developed for each mathematical concept in Montessori education. Abstract reasoning, such as mathematical quantities and relationships, is then developed through these lessons.

The Montessori *Children's House* provides children the opportunity to utilize appropriate language for objects and activities in the Montessori mathematics curriculum (Lillard, 2005). This idea outlines a key component to the Montessori experience. Cited research by Lillard (2005) established a theme that children have difficulty identifying objects when they are presented as toys and a symbol. Therefore, it is suggested that the implementation of Montessori lessons as teaching materials and not a toy is a key component. Therefore, Montessori developed foundational didactic materials for teaching numerosity: (1) a series of cards with sandpaper figures (1, 2, 3, etc.); (2) a series of large cards bearing the same figures in smooth paper for the enumeration of numbers above ten; and (3) two boxes with small sticks for counting (Montessori, 1964).

Montessori short bead stairs. Initially, children are introduced to a concrete tool that introduced place value in the decimal system. This system is known as the *Golden Beads*. Dr. Montessori developed this educational material to illustrate numbers. Using the Golden Beads, numbers one through nine are represented with individual units. Although children are introduced to these materials and can count the beads, children often do not understand the one-to-one correspondence with the Golden Beads alone (Glermaine, 2008). Montessori developed the *Short Bead Stair* materials to help children grasp the idea of quantities. Short Bead Stairs are “a set of colored glass beads, in which each quantity is represented by the appropriate number of individual beads wired together as a bar with a specific, easily recognizable color” (Glermaine, 2008, pg. 19). Each number in the set is represented by a specific color: “1” by one red bead; “2” by two green beads connected together; “3” by three pink beads, and so forth (*see* Figure 1).

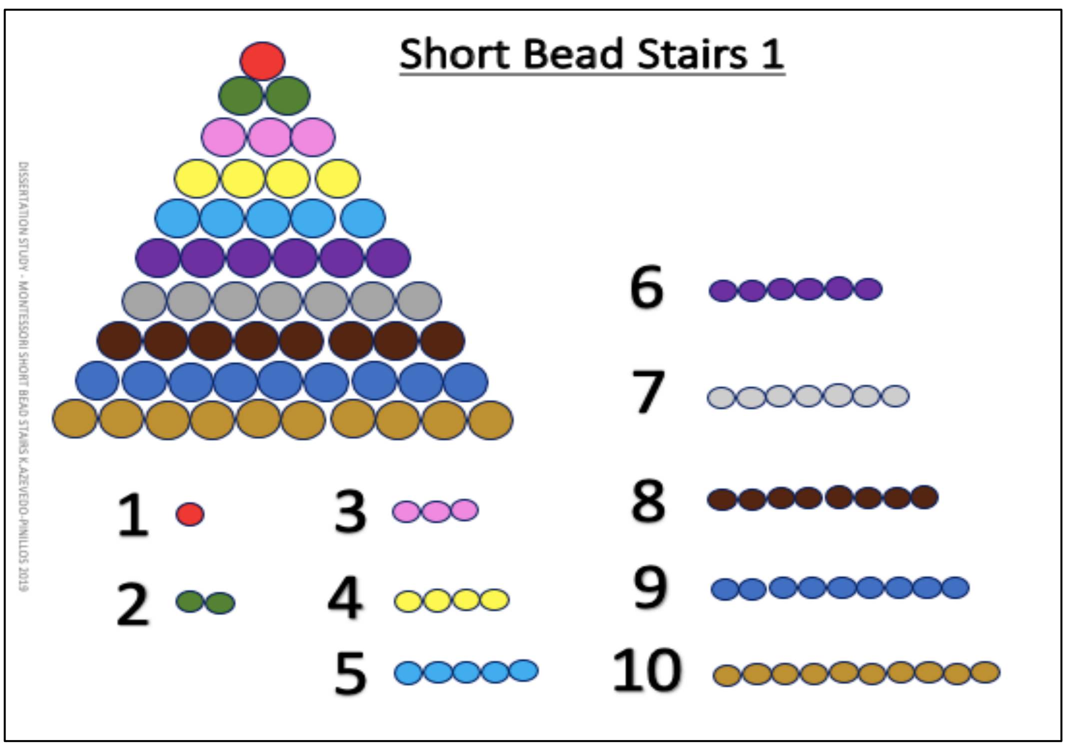


Figure 1. Short Bead Stairs with color coordination.

Initially, the Short Bead Stairs are utilized to count numbers one through nine using the Short Bead Stairs, a worksheet that illustrates the Short Bead Stairs, and number cards (*see* Figures 2 and 3) and provided. These beads are then continued to count and understand the quantity of all numbers, adding and subtracting, and eventually multiplication and division skills.

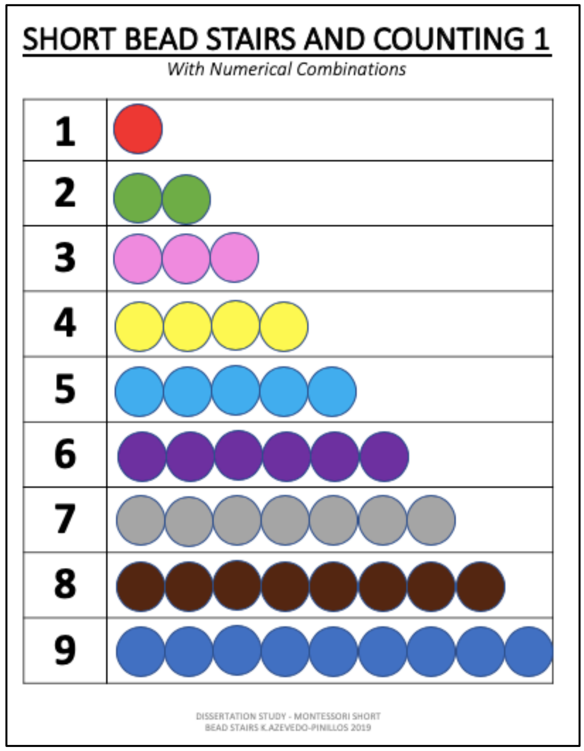


Figure 2. Montessori Short Bead Stairs worksheet with numerical combinations.

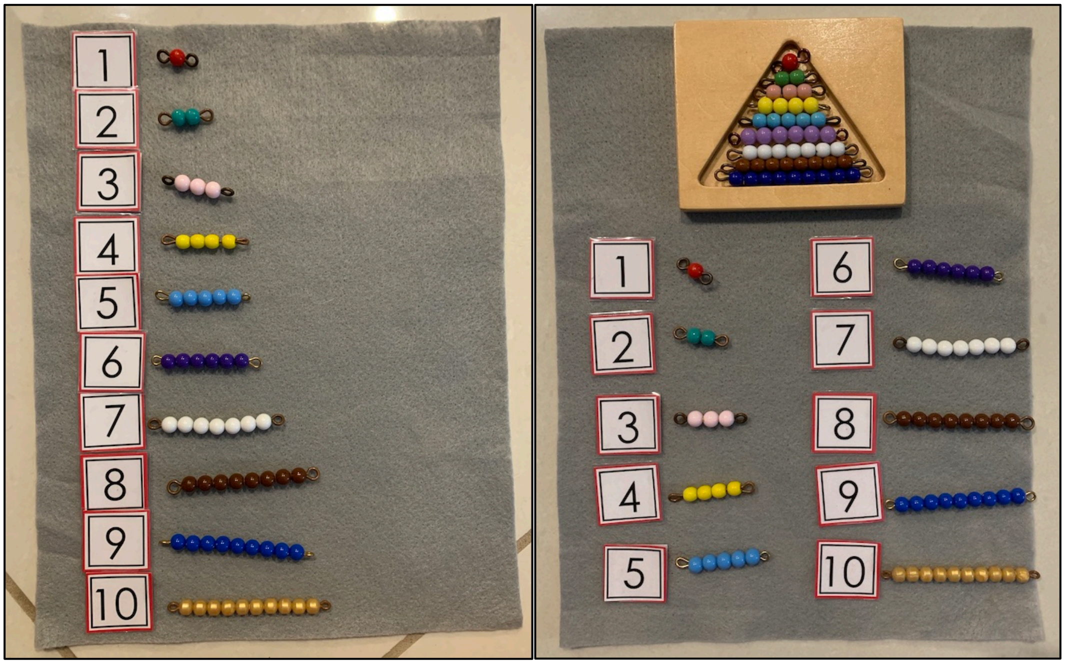


Figure 3. Demonstration of Montessori Short Bead Stairs.

Montessori teen boards. Once children have grasped the use of the Montessori Short Bead Stairs, children continue on to a different set of boards that introduce numbers greater than nine (Glermaine, 2008). This material is known as the *Montessori Teen Boards*. Teen Boards consist of two boards laid in a vertical row with numbers 10 through 19 listed. The boards are divided in nine sections with thin frames so children can insert wooden cards. The cards are numbered 1 through 9 and will cover one of the two digits listed. Children use the Short Bead Stairs in conjunction with the Teen Boards to illustrate and recognize the quantity of numbers.

Significance of the Study

There is a lack of empirical research on the effectiveness of the use of Montessori Short Bead Stairs, as well as the effectiveness of Montessori Methodology in mathematics for young children with disabilities. Empirical research is needed to investigate the specific techniques that affect the development of early numeracy skills intended to close the gap for school-age children who could potentially develop a learning disability in mathematics or struggle with mathematics later in education. Purpura and Lonigan (2015) compared 393 preschool children and the measurement of early numeracy skills (one-to-one counting, cardinality, counting subsets, subitizing, number comparisons, set comparison, number order, numeral identification, set-to-numerals, story problems, number combinations, and verbal counting). This study identified the need to efficiently assess children's numeracy skills to recognize individual instructional needs and measure student progress. Due to the lack of empirical data in using Montessori Short Bead Stairs, it is important to identify if this Montessori approach is effective in developing early numeracy skills and concepts in preschool-aged typically developing children and children with developmental delays.

Summary of Literature Review

In conclusion, Bauch and Hsu (1988) investigated comparisons between Montessori and Piagetian theories in relation to number concepts. Montessori utilized concrete materials, similar to those of Piaget, with a feedback component. However, Bauch and Hsu (1988) concluded that these tools and teaching methods are ageless and have demonstrated sufficiency compared to children using Piagetian serration tasks in traditional preschools. Yet, both theories have influenced the child's development of early numeracy skills and concepts. As early numeracy development consists of the counting principles (one-to-one correspondence, stable order, cardinality, sequencing of numbers, and basic number operations), this study focused on these early numeracy skill developments in children with and without developmental delays.

The above literature review also identified the early childhood programing, data and achievement, followed by special education categories and early intervention services and remediation. The focus on mathematical skill development of children identified the learning environment, mathematical tools, classroom discourse, mathematical tasks, assessment, and gender that influence mathematics growth in children with and without disabilities. With a focus on constructivism, Piaget's theory of development, and Montessori's Planes of Development, this theoretical framework served as the foundation for this study; more specifically, the teacher served as a facilitator of children's learning (Brooks & Brooks, 1999; Capriora & Angehilde, 2016), students were engaged in hands-on concrete tools (Roth, 2017; Steffe, 2016), the child's stages of development explained how preschool children processed numbers (Cakiroglu & Taskin, 2016; Navarro, 2014), and the effect of experiential learning on the development of mathematical knowledge (Caprioara & Angehilde, 2016). This study highlighted the major components of how children learn according to Montessori: children embrace hands-on learning from the world around them; children need less teacher direction and use their absorbent mind to engage with sensorial materials in a prepared environment; and children use their ability to retain and learn new skills during their sensitive period (Montessori, 1967; Pickering 1992). This literature review also compared and contrasted traditional and Montessori mathematical curricula and examined the impact of Montessori curricula on children with and without disabilities.

CHAPTER III

METHODOLOGY

This study examined the differential effects in student learning when the Montessori Short Bead Stairs (SBS) instruction was used compared to traditional approaches to counting, identifying, ordering, and identifying the quantity of numbers 1-10 in preschool students with and without developmental delays. Findings of this study may be beneficial for traditional teachers as they design instruction to effectively meet the needs of students with and without developmental delays. The study also surveyed participant teachers who implemented one of the teaching methods (Montessori SBS or Traditional) in their classroom. The survey focused on the teachers' perception of their reliability of administering daily lessons, confidence in teaching the interventions, willingness to utilize the lessons after the study completion, and their perception of how well students understood the lessons presented.

Chapter 3 describes the research methodology used in the study and provides a description of the overarching research question investigated, the design of the study, and participants who were involved. In addition, this chapter presents ethical considerations, data collection, procedures, and proposed data analyses.

Research Design

This study consisted of a randomized alternative-treatment with pretest design to investigate the difference in performance of students, with and without developmental delays, who used Montessori SBS and those taught by the Traditional non-Montessori methods in counting, identifying, ordering, and identifying the quantity of numbers 1-10. Classes were randomized to treatment conditions (Montessori SBS vs. Traditional methods; Shadish, Cook, & Campbell, 2002). Given that some of the variance may be explained by the various demographic variables, procedures were used to determine whether any differences existed across gender, ethnicity, age, and developmental delay. Because the study compared different substantive treatments, the alternative-treatments design with pretest was used to reduce the plausibility of alternative explanations and to facilitate causal inferences (Shadish et al., 2002).

R	O	X_A	O
R	O	X_B	O

Figure 4. Alternative-treatments design with pretest. Adapted from "Experimental and Quasi-Experimental Designs for Generalized Causal Inference," by W.R. Shadish et al., 2002, p. 258.

In this randomized alternative-treatment with pretest design, 16 classroom teachers and their classes from three preschools were randomly assigned to one of two treatment groups: Traditional or Montessori SBS. In a three-week intervention, 159 preschool students were assessed for pre- and post-test skills using subsections of the Test of Early Mathematics Ability (3rd edition) Form A. All participating classes from each school were randomly assigned to a treatment group (Montessori SBS vs. Traditional). Each teacher received training in the method to which she was assigned. In both interventions, for a minimum of 10 minutes every day for three weeks, students were instructed using activities from one of the two intervention methods. After three weeks of the intervention methods, students were once again assessed (post-test) using the same subsections of the TEMA-3 Form B. At the conclusion of the intervention and post-tests, teachers were then surveyed. The survey focused on the teachers' perceptions of their reliability of administering daily lessons, confidence in teaching the interventions, willingness to utilize the lessons after the study completion, and their perceptions of the degree to which students understood the lessons presented.

Participants and Sampling

The sample consisted of students and teachers at three non-Montessori preschools in a small, urban community located in southeast Florida, that had a combined student population of 1,551 preschool-aged students (3-5 years old). There were approximately 19 Voluntary Prekindergarten (VPK) providers in the area. The three schools who participated in this study were among the 19 VPK providers. The population of the neighborhood, as reported in the United States Census (2018), consisted of predominantly middle-class Hispanics (43%) and White (30.4%), with African Americans (20.4%), Asian (5.5%), and Other (0.7%).

To recruit participants for the study, the principal investigator (PI) contacted the directors of three private schools regarding the research study and sample size; each director expressed an interest in the study. There were three preschool classrooms from School #1, six preschool classrooms from School #2, and three preschool classrooms from School #3 that agreed to participate for a total of 12 participating classes. These private schools accept children with and without developmental delays.

Teacher participants. Sixteen female preschool teachers from three schools were invited to participate in this study. No further demographic data for teacher participants were obtained. Teachers implemented the interventions in their respective classroom; however, only 13 (81%) of the total teachers completed the survey. Of the 13 teachers who responded to the survey, eight of the teachers (62%) provided the Traditional instructional method, and five teachers (38%) provided the Montessori instructional method (SBS).

Student participants. Of the 184 students enrolled in the 16 classrooms participating in this study, 161 parents provided consent for their child to participate. Of the 161 participating students, two of the students declined to participate, yielding a total of 159 participants. Student characteristics of the resultant sample (N = 159) can be seen below in Table 1. The table offers a comparison of both treatment groups. Subsequent to random assignment, 53.5% of all participants (n = 85) received interventions using the Montessori SBS, and 46.5% of all participants (n = 74) received interventions using a Traditional method.

Table 1 demonstrates the percentage of children in each treatment group by ethnicity, age, gender, and developmental delay. Participants in both treatment groups were predominantly Hispanic. Children ranged from ages 3 to 5 years of age. The majority of participants in the Traditional group were age 3, while the majority of participants in the Montessori group were age 4. The study included a similar number of males (50.9%) as compared to females (49.1%) overall. Similarly, there were 37 males and 37 females who participated in the Traditional group. In the Montessori group, there were 44 males and 41 females who participated in this study.

Students with developmental delays were identified based on one of the following criteria: (a) child was currently receiving speech, language, physical, or occupational therapy; (b) the teacher had identified the child as working significantly below his/her peers; or (c) this researcher, with expertise in exceptional student education, identified the child as presenting characteristics of a child with a developmental (mentally or physically) delay. Thirty-five percent (*n* = 57) of the preschool students who participated in this study were identified as having a developmental delay. Both treatment groups included more students without developmental delays, compared to their peers who had been identified as having the characteristics of developmental delay.

Table 1

Student Ethnicity, Age, Developmental Delay, and Gender by Treatment Groups

	Traditional	Montessori SBS	Total
Characteristic	*n*	*n*	*n*
Ethnicity			
African American	14	13	27
Asian	6	18	24
Hispanic	30	23	53
Other	6	11	17
White	18	20	38
Age			
3-Years-Old	43	21	64
4-Years-Old	21	42	63
5-Years-Old	10	22	32
Gender			
Male	37	44	81
Female	37	41	78
Developmental Delay			
Non-Developmental Delay	48	54	102
Developmental Delay	26	31	57

Note. Participants were randomly assigned by class to the Traditional treatment group (n = 74) or Montessori SBS treatment group (n = 85). (N = 159)

Ethical Considerations

Prior to implementation, permission to conduct this study was obtained from each private school director where the study took place. In addition, the research obtained approval from the Institutional Review Board (IRB) of Barry University. Using a Participant Demographic Form, the school director from each school provided student-level information that included the first names and last name initial of each student, the child's age, ethnicity and any documented developmental delay for the purpose of linking student data (see Appendix A). An identification number was assigned to each student and data were recorded on a student data sheet by the director of each school to protect the identity of the children. Completed participant demographic forms were then provided to the researcher.

To protect student privacy, a number was assigned to each child and student names were omitted from all assessment sheets. Throughout the study, all student data sheets with student names were kept separate from the study data (pre and post-tests) and were secured in a locked cabinet. Adhering to Barry University's institutional policy, the data will be kept for a period of five years following completion of the study. All presentations or publication of the findings will be reported as aggregate data. No student names will ever be reported.

Instrumentation

Test of Early Mathematics Ability – Third Edition (TEMA-3). The TEMA-3 is a clinical assessment to that is privately-owned by Pro-Ed. Pro-Ed has an application process that allowed this researcher to request a free test kit for the dissertation/thesis purposes. The researcher was required to complete an examiner's qualification form that indicated the researcher's educational background, professional credentials, and testing and measurement coursework to receive approval for using and administering the assessment for this study. The researcher's advisor (dissertation chairperson) also signed the application for approval. The researcher requested and was granted one free test kit for use in this study. The TEMA-3 has two forms (Form A and B) that were provided and used in this study.

The original TEMA was developed by Ginsburg and Baroody in 1983. The TEMA-3 is the most updated version of the assessment (Ginsburg & Baroody, 2003). This clinical instrument measures the mathematical performance of children ages 4 through 8 and is helpful in identifying learning deficits in older children. This norm-referenced measure identifies strengths and weaknesses, progress, informal and formal concepts and skills, such as number skills, number literacy, mastery of number facts and calculation skills, across a total of 72 items (Ginsburg & Baroody, 2003). Counting, number identification, writing numbers, comparison of numbers, and sorting are also assessed in addition to arithmetic skills mentioned above (Papadakis, Kalogiannakis, & Zaranis, 2017; Van Herwegen, Costa, Nicholson, & Donlan, 2018). Additionally, quality comparisons, one-to-one correspondence counting, and oral counting are measured using the TEMA-3 (Green, Gallagher, & Hart, 2018). Assessment with the TEMA-3 involves the test administrator presenting a set of trials and/or questions that test age-appropriate skills to the preschool student.

Internal consistency reliabilities of above 0.92 have been reported for the TEMA-3 (Green et al., 2018; Papadakis et al., 2017; Purpura et al., 2015; Van Herwegen et al., 2018). Form A of the TEMA-3 was used as a pretest and Form B was used for the post-test (see Appendix B for TEMA-3 Permission Form; Mattoon et al., 2015). Research on preschool children suggests that it is better to use a brief broad content screening tool (vs. the full battery) that measures discrete skill measures for the purpose of identifying children at risk of later difficulties (Foegen, Jiban, & Deno, 2007; Purpura et al, 2015). Consequently, from the TEMA-3, 26 easy-to-administer items were identified as measuring early numeracy skills across a range of ability and were used to identify early numeracy development skills for children in preschool for this present study (Purpura et al., 2015). These skills include: verbal counting, counting forward or backward, counting error identification, one-to-one counting, cardinality, resultative counting, counting a subset, subitizing, estimation, ordinality, relative size, number comparison, set comparison, number order, sequencing set reproduction, numeral identification, set to numerals, addition or subtraction with objects, story problems, initial equivalence, two-set addition or subtraction, equivalent sets, number composition and decomposition, and number combinations.

Purpura et al. (2015) conducted a study of 393 preschool children that targeted specific components of early numeracy skills using the subset of 26 items drawn from the TEMA-3 without reducing the reliability of the measure. This subset of items was tested by Purpura et al. (2015) for sensitivity to age-related differences in performance and to establish cutoff scores for identifying children at risk of mathematics difficulties. These 26 specific skill questions from Purpura et al. (2015) were used for the current study. Identical skill-type questions were presented on both forms of the TEMA-3; thus, the pre- and post-tests were parallel forms. More specifically, the pre and post assessments included a total of 26 questions: five questions related to counting; five questions related to identifying numbers; six questions related to ordering numbers; and ten questions related to identifying the quantity of numbers.

Table 2 demonstrates the number and percentage of questions of each skill on the TEMA-3 forms A and B. Beginning with counting numbers and ending with identifying the quantity of numbers 1-10, the complexity of questions increased throughout the assessment. Moreover, most questions were related to identifying the quantity of numbers 1-10. Each question had two to five trials. To achieve a score for the question, the child needed to answer a certain number of trials correctly. Each question had a different number of trials needed to achieve the score criteria. If the child met the score criteria, he/she was awarded a point for the question. If he/she did not meet the score criteria, no points were awarded. For example, question #1 required the child to correctly identify how many cats he/she saw in the Picture Book (trial 1 = 3 cats; trial 2 = 1 cat; trial 3 = 2 cats). The child was required to correctly respond to all three trials in order to receive one point for the question. Scores were calculated by percentage of correctly answered questions (counting, identifying numbers, ordering numbers, and identifying the quantity of numbers).

Table 2

TEMA-3 Number and Percentage of Numeracy Skills Questions for Pre- and Post-Tests

Numeracy Skill	Questions	%
Counting Numbers 1-10	5	19.2
Identifying Numbers 1-10	5	19.2
Ordering Numbers 1-10	6	23.1
Identifying Quantity of Numbers 1-10	10	38.5

Note. Test of Early Mathematics (3rd ed.) Forms A and B included the four skill sets (Counting Numbers, Identifying Numbers, Ordering Numbers, Identifying the Quantity of Numbers).

Teacher Survey Instrument. The teacher survey consisted of 16 questions about teachers' perceptions of their confidence and reliability of teaching the interventions in the classroom (see Appendix C). The survey was conducted at the end of the study and was made available to the teachers online via Survey Monkey using a Likert Rating Scale. The Likert Scale is a type of rating scale used to measure the attitudes or opinions of the respondents based on a level of agreement: (1) strongly disagree; (2) disagree; (3) neutral; (4) agree; (5) strongly agree (Harpe, 2015). Questions were grouped into one of the following categories: (a) reliability of administering the daily lessons of the intervention; (b) confidence in teaching the interventions (c) willingness to utilize the lessons after the study completion; and (d) whether students understood the lessons presented.

Procedures

In this randomized alternative-treatment with pretest design, the PI randomly assigned each class to one of two treatment groups: Traditional or Montessori. All student participants were administered the TEMA-3 (Form A) as a baseline assessment (individually by the PI or by a research assistant who was trained for the task) received three weeks of intervention (by the participating classroom teacher) and were administered a post-test of the TEMA-3 (individually by PI or the research assistant). Teacher surveys were conducted at the completion of the study via Survey Monkey.

Consent/Assent Procedures

Teacher consent forms. Each school director served as a gatekeeper and emailed an invitation letter and consent form to each teacher (see Appendices D & E). The school directors then collected the returned signed consent forms and provided the informed consents to the PI.

Parent consent forms. Directors from each participating school posted recruitment flyers and provided a copy of the consent form to all parents of children in the classrooms of teachers who consented to participate (Appendices F & G). The director collected all signed parent consent forms, arranged them according to class, placed them in an envelope provided by the PI, and returned the envelope to the PI. No assent forms were provided as children in this study were too young; however, before the initial baseline data was collected, the teacher discussed the study (at the child's level of understanding) with the students and children verbally accepted or refused to participate in the study. Two students refused to participate. Students who declined to participate continued with the Montessori or Traditional interventions provided by the classroom teacher. However, pre- and post-test data were not collected for students who declined to participate. Students whose parents did not consent to the study, were not administered the pre- and post-tests of the TEMA-3. Once consent was obtained, each participant was assigned a number for the purpose of confidential data collection and reporting.

Modification to Research Study

Subsequent to the start of the study, the researcher revisited the need for additional assistance in efficiently administering the post assessments. The IRB approved a modification that included a change in the study personnel to include the assistance of another educator in the administration of the pre- and post-tests of the TEMA-3.

Training

After obtaining the consent forms from the director of each school, the PI randomly assigned each teacher (and her class) to a treatment group (Montessori SBS vs. Traditional). The PI placed one teacher and her class into the Montessori group, the next into the Traditional group, and assigned each subsequent class in that order to one of the two treatment groups. The PI conducted two 2-hour training sessions per school (one on Montessori SBS and one on traditional, non-Montessori methods). Teachers only attended the training associated with the intervention to which they had been assigned.

The training included an overview of the study, a detailed explanation about the assigned intervention process, and opportunities for the teacher to practice the intervention before implementing it in his/her own classroom. Each training consisted of individualized, direct explanations, and practice of the treatment. Teachers who used Traditional interventions were provided traditional worksheets and training materials; while teachers who used the Montessori instruction were provided Montessori SBS training materials and worksheets with accompanying didactic materials (see Appendices H & I).

Following the training, each teacher implemented the appropriate intervention in the classroom for three weeks. The PI visited each school two times during the implementation of the interventions to confirm that the interventions were being implemented correctly and to answer any questions from the teachers. Fidelity of the intervention was achieved through training and monitoring.

Baseline Data /Pre-Test

Subsequent to teacher training and the collection of consent forms, Form A (26 subset questions) of the Test of Early Mathematics Ability – Third Edition (TEMA-3) was administered too all participants for baseline data. Pre-test assessments varied in the length of time it took students to complete the items. The time to complete pre-test assessments ranged from 10 to 35 minutes depending on the child's level of knowledge, attentiveness, and processing.

Intervention

Students in the Traditional group learning mathematical counting using traditional methods (e.g., rote memorization, counters, blocks) were provided worksheets. Students in the Montessori group received the intervention of the Montessori SBS that included Montessori SBS didactic materials and worksheets. All groups used the appropriate intervention approximately ten minutes each day for three weeks.

Post-Test

Subsequent to the three weeks of intervention, all students from both treatment groups were assessed for post-intervention results using Form B of the TEMA-3 during the last week of the study. The TEMA-3 Form B subset questions were administered to each child participant individually. Post-test assessments varied in time length; it took approximately 10 to 35 minutes per student depending on the student's level of knowledge, attentiveness, and processing.

Teacher Survey Procedures

Upon completion of the post-tests, each participating teacher was emailed a Teacher Survey via SurveyMonkey to complete. The survey included questions about the teachers' perception of her reliability of administered daily lessons, confidence in teaching the interventions, willingness to utilize the lessons after the study completion, and whether students understood the lessons presented.

Data Collection

Students were assigned a number for the purpose of confidential data collection and reporting. A Participant Demographics Form was used to collect data from school directors regarding student demographics. A separate Data Collection Form was used to collect pre- and post-test results (see Appendix J). Participant names were kept in a location separate from the data to protect confidentiality. The PI maintained an exact list of participant names and numbers to ensure data collection was accurate.

Data Analysis

For each of four dependent variables, 2 x 2 analyses of variance (ANOVA) were conducted on pre- and post- tests. Follow-up *t* tests were conducted to test the relationships between the pre- and post-tests. Eight 2 x 2 analyses of variance (ANOVAs) were conducted to assess whether differences existed in student's ability to count, identify, order, and identify the quantity of numbers 1-10 on the pre- and post-tests of students placed in the Traditional group as compared to students in the Montessori group. Interaction differences were also assessed to determine the effects between developmental delay status and overall performance. Each 2 x 2 ANOVA was suitable for this study and can be seen in Figure 5. To report teacher survey findings, frequencies were analyzed for each question to determine teachers' responses by percentage. The Statistical Package for the Social Sciences (SPSS) on Mac, Version 23 (2015) was used to analyze the data and prepare charts for the purpose of data reporting.

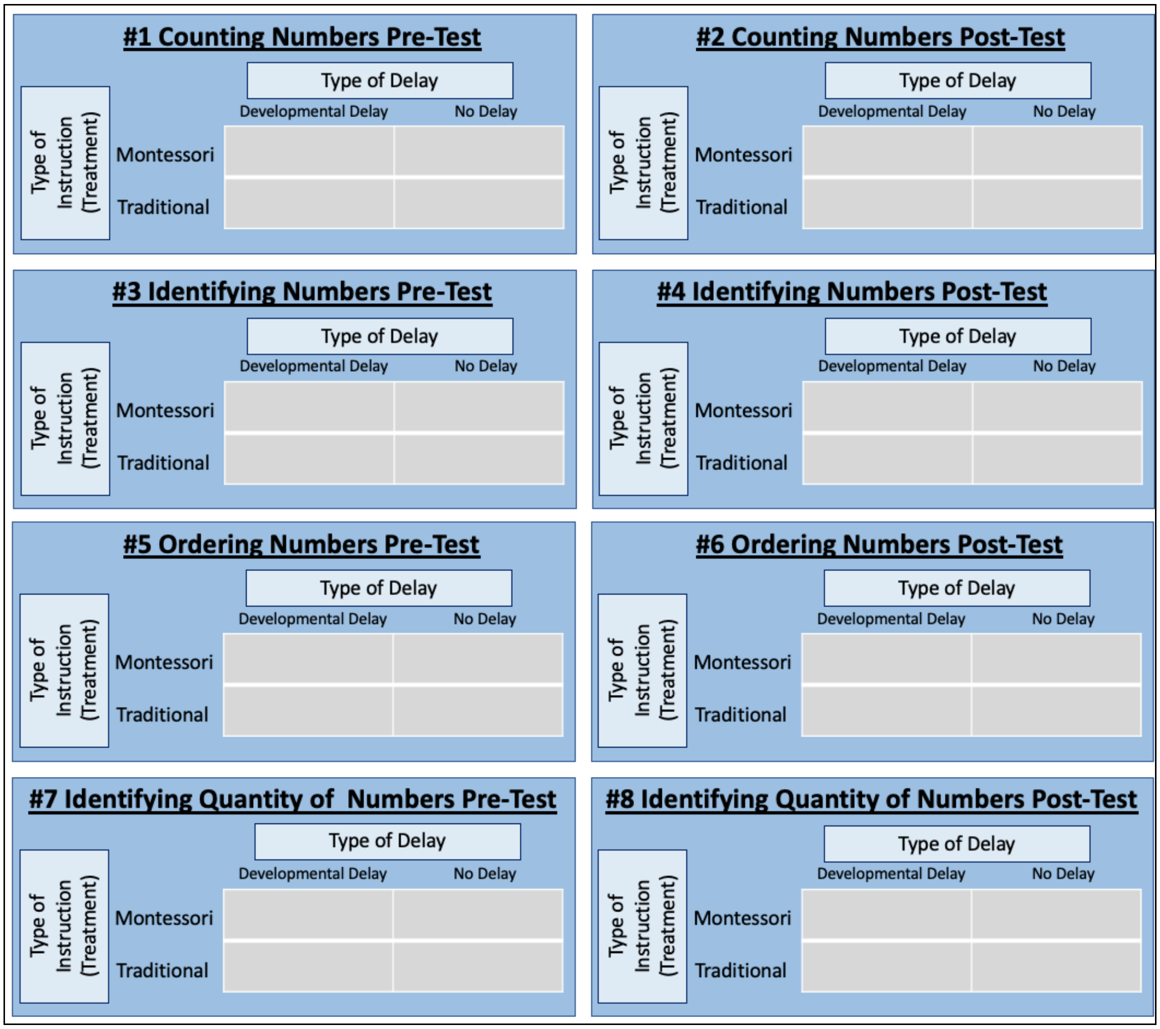


Figure 5. Eight 2 x 2 analyses of variance (ANOVAs) designs. Each dependent variable (counting numbers, identifying numbers, ordering numbers, and identifying the quantity of numbers) was assessed for differences between the independent variables (delay and type of instruction groups) on the pre- and post-tests.

Summary

Chapter III described the research methodology used in the study and provided a description of the overarching research questions, study design, and participants. Additionally, ethical considerations, data collection procedures, instrumentation, and data analysis procedures were addressed. Chapter IV addresses the results of the study and Chapter V explains the findings, limitations, and recommendations for further study.

CHAPTER IV

RESULTS

The following section presents the results of tests related to each of the hypotheses in this study. By reporting the effect size, exact *p* values, and confidence levels, the analysis maintained its power and limited Type II errors (Nakagwa, 2004; Perneger, 1998). These results provide the foundation for the discussion in Chapter V.

Hypothesis #1 - Counting Numbers 1-10

Hypothesis 1 investigated the performance of students in counting numbers separately for Developmental Delay and Non-Developmental Delay groups by treatment groups (Traditional vs. Montessori SBS). To test this hypothesis, two 2 x 2 factorial ANOVAs were conducted. Follow-up *t* tests were run to investigate changes in the pre- and post-tests for each treatment and developmentally delay group.

Test of Assumptions for Counting Numbers 1-10

To test the hypothesis that performance in counting numbers 1-10 would be related to the type of intervention and presence or absence of a developmental delay (H_a1), two 2-way analyses of variance (ANOVAs) were conducted. Additional paired groups *t* tests were conducted to asses pre/post differences. A preliminary analysis of the assumption that the homogeneity of variance for the dependent variables had not been violated was tested using Levene's test, which indicated that the error variance of counting numbers 1-10 was not equal across groups on the pre-test, $F(3, 155) = 8.79$, $p < .001$; however, the error variance of counting numbers 1-10 was equal across groups on the post-test, $F(3, 155) = 1.66$, $p = .178$. That is, the assumption of homogeneity was violated for the pre-test, but not the post-test.

ANOVAs for Counting Numbers 1-10 by Treatment and Delay Groups

Descriptive statistics for counting numbers 1-10 can be seen in Table 3. For comparison, data in the column totals depict mean pre-test and post-test scores for Developmental Delay and Non-Developmental Delay groups collapsed across teaching methods. Similarly, row totals depict pre-test and post-test means for the two teaching methods collapsed across Developmental Delay/Non-Developmental Delay groups.

Table 3

Descriptive Statistics for Counting Numbers (1-10) by Developmental Delay Groups and Treatment Group (N = 159)

		Traditional			Montessori SBS			Total		
DD		*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*
ND	Pre-Test	60.42	23.15	48	64.07	28.12	54	62.35	25.84	102
	Post-Test	75.21	23.97	48	77.04	25.00	54	76.18	24.42	102
DD	Pre-Test	29.23	26.07	26	43.23	37.27	31	36.84	33.12	57
	Post-Test	51.54	31.07	26	61.94	30.29	31	57.19	30.81	57
Total	Pre-Test	49.46	28.33	74	56.47	33.12	85	53.21	31.08	159
	Post-Test	66.89	28.81	74	71.53	27.84	85	69.37	28.30	159

Note. Pre- and post-test scores ranged from a minimum of zero to a maximum of 100. ND = non-developmental delay. DD = developmental delay.

Interaction between treatment and delay groups for counting numbers 1-10. Two 2 x 2 (Type of Intervention x Developmental Delay) factorial ANOVAs were conducted on the performance of counting numbers 1-10 separately for pre-test and post-test. As can be seen in Table 4, there was no significant interaction between

Developmental Delay vs. treatment groups (Traditional vs. Montessori SBS) on the pre-test, $F(1, 15) = 1.20$ $p = .276$, partial $\eta^2 = .01$, nor post-test, $F(1, 155) = .93$, $p = .338$, partial $\eta^2 = .01$. Because the interaction effect was not significant, the main effects are reported.

Table 4

Hypothesis #1: Two 2-Way Analyses of Variance of Pre- and Post-Test Counting Numbers (1-10) Scores with Developmental Delay and Type of Instruction as Fixed Factors

	Pre-Test				Post-Test			
Source	*df*	*MS*	*F*	*p*	*df*	*MS*	*F*	*p*
DD	1	24598.15	30.32	< .001	1	13656.80	18.94	< .001
Type of Instruction	1	2830.97	3.49	.064	1	1357.92	1.88	.172
DD *Type of Instruction	1	970.88	1.20	.276	1	666.97	.93	.338
Error	155	811.34			155	720.94		
Total	159				159			

Note. DD = developmental delay.

Main effects for counting numbers 1-10 for treatment. There was no significant main effect for type of instruction on either the pre-test, $F(1, 155) = 3.49$, $p = .064$, partial $\eta^2 = .02$, nor on the post-test, $F(1, 155) = 1.88$, $p = .172$, partial $\eta^2 = .01$. These results suggest that the type of instruction (Montessori SBS or Traditional) did not differentially impact a child's performance on counting numbers 1-10.

Main effects of counting numbers 1-10 for delay. On the other hand, there was a significant main effect for Developmental Delay for both the pre-test, $F(1, 155) = 30.32$, $p < .001$, partial $\eta^2 = .16$, and post-test, $F(1, 155) = 18.94$, $p < .001$, partial $\eta^2 = .11$. These results show that students identified as having a developmental delay performed more poorly on both pre- and post-tests for counting numbers 1-10, compared to students without developmental delays.

Paired Samples *t* Tests to Assess Pre/Post Differences in Counting Numbers 1-10 by Treatment and Delay

To explore pre/post differences within treatment and developmental delay groups, four paired samples *t* tests were conducted to compare the performance of counting numbers 1-10 before and after the intervention for both treatment groups (Traditional vs. Montessori SBS) and developmental delay groups (Developmental Delay vs. Non-Developmental Delay) collapsed across groups.

Treatment groups/Counting Numbers 1-10. Two paired samples *t* tests were conducted to compare the performance of counting numbers 1-10 before and after the intervention for treatment groups collapsed across developmental delay groups. There was a significant increase in counting numbers 1-10 for students in the Traditional group from the pre-test ($M = 49.46$, $SD = 28.33$) to the post-test ($M = 66.89$, $SD = 28.81$); $t(73) = -8.88$, $p < .001$. Similarly, there was a significant increase in counting numbers 1-10 for students in the Montessori SBS group on the post-test ($M = 71.53$, $SD = 27.84$), compared to the pre-test ($M = 56.47$, $SD = 33.12$), $t(73) = -6.133$, $p < .001$. Results of paired samples *t* tests can be seen below in Table 5. These results suggest that both mathematical interventions in general, improve children's skills on counting numbers 1-

10. More specifically, these results suggest that when preschool children are provided Montessori SBS or Traditional lessons, learning to count numbers 1-10 will improve.

Table 5

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Counting Numbers (1-10) by Treatment Groups

Variable		*M*	*SD*	*SE*	95% Confidence Interval *LL*	95% Confidence Interval *UL*	*p*
Treatment							
	Traditional	-17.43	16.89	1.96	-21.34	-13.52	< .001
	Montessori SBS	-15.06	22.66	2.46	-19.95	-10.17	< .001

Note. LL = lower level. UL = upper level.

Delay groups/Counting Numbers 1-10. Similarly, two paired samples *t* tests were also conducted to compare performance on counting numbers 1-10 before and after the treatment for developmental delay groups (Developmental Delay vs. Non-Developmental Delay), collapsed across teaching methods. As seen below in Table 6, there was a significant increase in counting numbers 1-10 for students without developmental delays on the post-test (M = 76.18, SD = 24.42), compared to the pre-test (M = 62.35, SD = 25.84), $t(101)$, = -8.30, $p < .001$. Similarly, there was also a significant increase for students with developmental delays on the post-test (M = 57.19, SD = 30.81), compared to the pre-test (M = 36.84, SD = 33.12), $t(56)$ = -6.24, $p < .001$. These results suggest that children with and without developmental delays, when presented with

Montessori SBS or Traditional lessons, improved their mathematical skills in counting numbers 1-10.

Table 6

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Counting Numbers (1-10) by Developmental Groups

Variable		*M*	*SD*	*SE*	95% Confidence Interval		*p*
					LL	*UL*	
DD							
	No Delay	-13.82	16.83	1.62	-17.13	-10.52	< .001
	DD	-20.35	24.64	3.26	-26.89	-13.81	< .001

Note. LL = lower level. UL = upper level. DD = developmental delay.

Hypothesis #2 – Identifying Numbers 1-10

Hypothesis 2 investigated the performance of students on identifying numbers separately for developmental delay groups (Developmental Delay vs. Non-Developmental Delay) and treatment groups (Traditional vs. Montessori SBS). To test this hypothesis, two 2 x 2 ANOVAs were conducted. Follow-up *t* tests were run to investigate changes in the pre- and post-tests for each developmental delay and treatment group.

Test of Assumptions for Identifying Numbers 1-10

To test the hypothesis that performance in identifying numbers 1-10 would be related to the type of intervention and presence or absence of a developmental delay (H_a2), two 2-way analyses of variance (ANOVAs) were conducted. Additional paired groups *t* tests were conducted to asses pre/post differences. A preliminary analysis, Levene's Test, conducted to evaluate the homogeneity of variances, indicated that the error variance of identifying numbers was equal across groups on the post-test, $F(3, 155) = 1.55$, $p = .204$; however, the error variance of identifying numbers was not equal across groups on the pre-test, $F(3, 155) = 6.46$, $p < .001$.

ANOVAs for Identifying Numbers 1-10 by Treatment and Delay Groups

Descriptive statistics for identifying numbers 1-10 can be seen in Table 7. For comparison, data in the column totals depict mean pre-test and post-test scores for Developmental Delay and Non-Developmental Delay groups collapsed across teaching methods. Similarly, row totals depict pre-test and post-test means for the two teaching methods collapsed across Developmental Delay/Non-Developmental Delay groups.

Table 7

Descriptive Statistics for Identifying Numbers (1-10) by Developmental Delay Groups and Treatment Group (N = 159)

		Traditional			Montessori SBS			Total		
DD		*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*
ND	Pre-Test	30.00	30.04	48	35.19	32.95	54	32.75	31.57	102
	Post-Test	46.60	33.66	48	51.06	32.72	54	48.96	33.07	102
DD	Pre-Test	10.00	17.21	26	27.10	27.10	31	19.30	24.49	57
	Post-Test	20.77	25.60	26	36.77	33.11	31	29.47	30.73	57
Total	Pre-Test	22.97	27.83	74	32.24	31.03	85	27.92	29.85	159
	Post-Test	37.53	33.29	74	45.85	33.39	85	41.97	33.49	159

Note. Pre- and post-test scores ranged from a minimum of zero to a maximum of 100. ND = non-developmental delay. DD = developmental delay.

Interaction between treatment and delay groups for identifying numbers 1-10. Two 2 x 2 (Type of Intervention x Developmental Delay) factorial ANOVAs were conducted on the influence of independent variables for identifying numbers 1-10 on the pre- and post-tests. As can be seen in Table 8, the ANOVA did not reveal a significant interaction effect between Developmental Delay and type of instruction (Traditional vs. Montessori SBS) for identifying numbers on the pre-test, $F(1, 155) = 1.54$, $p = .216$, partial η^2 = .01, nor the post-test $F(1, 155) = 1.18$, $p = .276$, partial η^2 = .01. Because the interaction effect was not significant, the main effects are reported.

Table 8

Hypothesis #2: Two 2-Way Analyses of Variance of Pre- and Post-Test Identifying Numbers (1-10) Scores with Developmental Delay and Type of Instruction as Fixed Factors

	Pre-Test				Post-Test			
Source	*df*	*MS*	*F*	*p*	*df*	*MS*	*F*	*p*
DD	1	7167.70	8.59	.004	1	14620.66	14.23	< .001
Type of Instruction	1	4510.58	5.40	.021	1	3801.73	3.70	.056
DD *Type of Instruction	1	1289.04	1.54	.216	1	1212.71	1.18	.279
Error	155	834.75			155	1027.28		
Total	159				159			

Note. DD = developmental delay.

Main effects for identifying numbers 1-10 for treatment. A significant main effect for type of instruction existed on the pre-test, $F(1, 155) = 5.40$, $p = .021$ partial η^2 = .03, yet there was no significant main effect on the post-test, $F(1, 155) = 3.70$, $p = .056$, partial η^2 = .023. These results suggest that although difference approached significance on the post-test, because of the difference at the pre-test, it cannot be assumed that this difference was caused by the intervention. Therefore, the treatment did not make a difference for identifying numbers 1-10.

Main effects of identifying numbers 1-10 for delay group. There was a significant main effect for developmental delay on both the pre-test, $F(1, 155) = 8.59$, $p = .004$ partial $\eta^2 = .05$, and post-test, $F(1, 155) = 14.23$, $p < .001$, partial $\eta^2 = .08$. The results indicate that students without a developmental delay ($M = 48.96$, $SD = 33.07$) performed better on the pre- test and post-test, compared to students with a developmental delay ($M = 29.47$, $SD = 30.73$) for identifying numbers 1-10.

Paired Samples *t* Tests to Assess Pre/Post Differences in Identifying Numbers 1-10 by Treatment and Delay

To explore pre/post differences within treatment and developmental delay groups, four paired samples *t* tests were conducted to compare identifying numbers 1-10 before and after receiving the intervention for both treatment groups (Traditional vs. Montessori SBS) and developmental delay groups (Developmental Delay vs. Non-Developmental Delay) collapsed across groups.

Treatment groups/Identifying Numbers 1-10. Two paired samples *t* tests were conducted to compare identifying numbers 1-10 for both treatment groups collapsed across developmental delay groups. There was a significant difference in identifying numbers 1-10 for students in the Traditional group from the pre-test ($M = 22.97$, $SD = 27.83$) to the post-test ($M = 37.53$, $SD = 33.29$); $t(73) = -6.46$, $p < .001$. Similarly, there was a significant increase in identifying numbers 1-10 for students in the Montessori group on the post-test ($M = 45.85$, $SD = 33.39$), compared to the pre-test ($M = 32.24$, $SD = 31.03$), $t(73) = -5.20$, $p < .001$. Results of paired samples *t* tests can be seen below in Table 9. These results suggest that mathematical interventions have an effect on identifying numbers 1-10. More specifically, these results suggest that when preschool children are provided Traditional or Montessori SBS lessons, skills learning to identify numbers 1-10 will improve.

Table 9

Mean Difference Scores and Paired Samples t Test Results Compare to Pre/Post-Tests of Identifying Numbers (1-10) by Treatment Groups

Variable		*M*	*SD*	*SE*	95% Confidence Interval *LL*	*UL*	*p*
Treatment							
	Traditional	-14.55	19.38	2.25	-19.04	-10.06	< .001
	Montessori SBS	-13.61	24.14	2.62	-18.82	-8.41	< .001

Note. LL = lower level. UL = upper level.

Delay groups/Identifying Numbers 1-10. Similarly, two paired samples *t* tests were conducted to compare identifying numbers 1-10 before and after the treatment for both developmental delay groups collapsed across teaching methods. As depicted in Table 10, there was a significant increase in identifying numbers 1-10 for students without developmental delays on the post-test ($M = 48.96$, $SD = 33.07$), compared to the pre-test ($M = 32.75$, $SD = 31.57$), $t(101)$, -7.87, $p < .001$. There was also a significant increase for students with developmental delays on the post-test ($M = 29.47$, $SD =30.73$), compared to the pre-test ($M = 19.30$, $SD = 24.49$), $t(56) = -3.25$, $p = .002$. These results suggest that children with and without developmental delays, regardless of Traditional or Montessori SBS lessons, demonstrated an improvement in mathematical skills in identifying numbers 1-10.

Table 10

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Identifying Numbers (1-10) by Developmental Delay Groups

					95% Confidence Interval		
Variable		*M*	*SD*	*SE*	*LL*	*UL*	*p*
DD							
	No Delay	-16.22	20.81	2.06	-20.30	-12.13	< .001
	DD	-10.18	23.64	3.13	-16.45	-3.90	.002

Note. LL = lower level. UL = upper level. DD = developmental delay.

Hypothesis #3 – Ordering Numbers 1-10

Hypothesis 3 investigated the performance of students on ordering numbers separately for developmental delay groups (Developmental Delay vs. Non-Developmental Delay) and treatment groups (Traditional vs. Montessori SBS). To test this hypothesis, two 2 x 2 factorial ANOVAs were conducted. Follow-up *t* tests were run to investigate changes in the pre- and post-tests for each developmental delay and treatment group.

Test of Assumptions for Ordering Numbers 1-10

To test the hypothesis that performance in ordering numbers 1-10 would be related to the type of intervention and presence or absence of a developmental delay (H_a3), two 2-way factorial ANOVAs were conducted. Additional paired groups *t* tests were conducted to asses pre/post differences. Levene's Test was conducted to evaluate the assumption of homogeneity of variance across groups. Results indicated that the error variance of ordering numbers are equal across groups on the pre-test, $F(3, 155) = .26$, $p = .854$, and post-test, $F(3, 155) = .11$, $p = .953$.

ANOVAs for Ordering Numbers 1-10 by Treatment and Delay Groups

Descriptive statistics for ordering numbers 1-10 can be seen in Table 11. For comparison, data in the column totals depict mean pre-test and post-test scores for Developmental Delay and Non-Developmental Delay groups collapsed across teaching methods. Similarly, row totals depict pre-test and post-test means for the two teaching methods collapsed across Developmental Delay/Non-Developmental Delay groups.

Table 11

Descriptive Statistics for Ordering Numbers (1-10) by Developmental Delay Groups and Treatment Group (N = 159)

		Traditional			Montessori SBS			Total		
DD		*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*
ND	Pre-Test	48.10	28.48	48	45.43	30.49	54	46.69	29.45	102
	Post-Test	51.23	30.48	48	57.67	30.37	54	54.64	30.44	102
DD	Pre-Test	19.88	27.90	26	31.16	29.97	31	26.02	29.34	57
	Post-Test	28.35	31.11	26	40.58	29.24	31	35.00	30.46	57
Total	Pre-Test	38.19	31.19	74	40.22	30.90	85	39.28	30.96	159
	Post-Test	43.19	32.41	74	51.44	30.91	85	47.60	31.79	159

Note. Pre- and post-test scores range from a minimum of zero and a maximum of 100. ND = no developmental delay. DD = developmental delay.

Interaction between treatment and delay groups for ordering numbers 1-10. Two 2 x 2 (Type of Intervention x Developmental Delay) factorial ANOVAs were conducted on the performance of ordering numbers 1-10 separately for pre-test and post-test. As can be seen in Table 12, there was no significant interaction effect between the type of intervention for ordering numbers 1-10 on the pretest, $F(3, 155) = .2.05$, $p = .154$, partial $\eta^2 = .013$, nor on the post-test, $F(3, 155) = .33$, $p = .565$, partial $\eta^2 = .00$. These results suggest that there were no effects on student's performance of ordering numbers 1-10 for based on developmental delay group for neither Traditional nor Montessori SBS teaching methods.

Table 12

Hypothesis #3: Two 2-Way Analyses of Variance of Pre- and Post-Test Ordering Numbers (1-10) Scores with Developmental Delay Groups and Type of Instruction as Fixed Factors

	Pre-Test				Post-Test			
Source	*df*	*MS*	*F*	*p*	*df*	*MS*	*F*	*p*
DD	1	16397.59	18.99	< .001	1	14513.52	15.80	< .001
Type of Instruction	1	671.68	0.78	.379	1	3167.43	3.45	.065
DD *Type of Instruction	1	1769.21	2.05	.154	1	305.30	0.33	.565
Error	155	863.31			155	918.52		
Total	159				159			

Note. DD = developmental delay.

Main effects for ordering numbers 1-10 for treatment. There was no significant main effect for type of instruction for ordering numbers on neither the pre-test, $F(1, 155) = .78$, $p = .379$, partial $\eta^2 = .01$, nor on the post-test, $F(1, 155) = 3.45$, $p = .065$, partial $\eta^2 = .02$. These results suggest that there were no differential effects on student's performance of ordering numbers 1-10 for Traditional vs. Montessori SBS teaching methods.

Main effects of ordering numbers 1-10 for delay. Two 2 x 2 factorial ANOVAs revealed a significant main effect for developmental delay of ordering numbers on both the pre-test, $F(1, 155) = 18.99$, $p < .001$, partial $\eta^2 = .11$, and post-test, $F(1, 155) = 15.80$, $p < .001$, partial $\eta^2 = .09$. Although instruction had no effect on the ordering of numbers 1-10, these results indicate that there is a significant main effect for the developmental delay variable and the ordering of numbers 1-10. More specifically, these results show that students identified as having a developmental delay performed more poorly on both pre- and post-tests for ordering numbers 1-10, compared to students without developmental delays.

Paired Samples *t* Tests to Assess Pre/Post Differences in Ordering Numbers 1-10 by Treatment and Delay

To explore pre/post differences within treatment and developmental delay groups, four paired samples *t* tests were conducted to compare the performance on ordering of numbers 1-10 before and after the treatment for both treatment (Traditional vs. Montessori SBS) and developmental delay groups (Developmental Delay vs. Non-Developmental Delay) collapsed across groups.

Treatment groups/Ordering Numbers 1-10. Two paired samples *t* tests were conducted to compare performance on ordering of numbers 1-10 on the pre- and post-tests for treatment (Traditional vs. Montessori SBS) collapsed across developmental delay groups. A significant difference did not exist in ordering numbers 1-10 for students in the Traditional group from the pre-test ($M = 38.19$, $SD = 31.19$) to the post-test ($M = 43.19$, $SD = 32.41$); $t(73) = -2.83$, $p < .006$. However, there was a significant increase in ordering numbers 1-10 for students in the Montessori SBS group on the post-test ($M =$

51.44, *SD* = 30.91), compared to the pre-test (*M* = 40.22, *SD* = 30.90), $t(73) = -4.60$, $p < .001$. These results suggest that Montessori SBS interventions have an effect on ordering numbers 1-10. More specifically, these results suggest that when preschool children are provided Montessori SBS lessons, learning to order numbers 1-10 will improve. Results of the paired samples *t* tests can be seen below in Table 13.

Table 13

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Ordering Numbers (1-10) by Treatment Groups

					95% Confidence Interval		
Variable		*M*	*SD*	*SE*	*LL*	*UL*	*p*
Treatment							
	Traditional	-5.00	15.23	1.77	-8.53	-1.47	.006
	Montessori SBS	-11.21	22.46	2.44	-16.06	-6.37	< .001

Note. LL = lower level. UL = upper level.

Delay groups/Ordering Numbers 1-10. Similarly, two paired samples *t* tests were conducted to compare the ordering of numbers 1-10 on the pre- and post-tests for developmental delay groups (Developmental Delay vs. Non-Developmental Delay), collapsed across treatment groups. As seen below in Table 14, there was a significant increase in ordering numbers 1-10 for students without developmental delays on the post-test (*M* = 54.64, *SD* = 30.44), compared to the pre-test (*M* = 46.69, *SD* = 29.45), $t(101)$, -4.05, $p < .001$. There was also a significant increase for students with developmental delays on the post-test (*M* = 35.00, *SD* = 30.46), compared to the pre-test (*M* = 26.02, *SD*

= 29.34), $t(56) = -3.49$, $p < .001$. These results suggest that preschool children with developmental delay improved their ability to order numbers 1-10 from the pre-test to post-test through the use of interventions.

Table 14

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Ordering Numbers (1-10) by Developmental Delay Groups

Variable		*M*	*SD*	*SE*	95% Confidence Interval *LL*	*UL*	*p*
DD							
	No Delay	-7.95	19.81	1.96	-11.84	-4.06	< .001
	DD	-8.98	19.42	2.57	-14.14	-3.83	< .001

Note. LL = lower level. UL = upper level. DD = developmental delay.

Hypothesis #4 – Identifying the Quantity of Numbers 1-10

Hypothesis 4 investigated the performance of students on counting numbers separately for developmental delay (Developmental Delay vs. Non-Developmental Delay) and treatment (Traditional vs. Montessori SBS) groups. To test this hypothesis, two 2 x 2 factorial ANOVAs were conducted. Follow-up *t* tests were run to investigate changes in the pre- and post-tests for each developmental and treatment group collapsed across groups.

Test of Assumptions for Identifying the Quantity of Numbers 1-10

To test the hypothesis that performance in identifying the quantity of numbers 1-10 would be related to the type of intervention and presence or absence of a developmental delay (H_a4), two 2-way analyses of variance (ANOVAs) were conducted. Additional paired groups t tests were conducted to asses pre/post differences Levene's Test was used to evaluate the homogeneity of variance and showed that variances for identifying the quantity of numbers was not equal across groups on the pre-test, $F(3, 155) = 5.81$, $p < .001$, nor post-test, $F(3, 155) = 3.02$, $p = .031$.

ANOVAs for Identifying the Quantity of Numbers 1-10 by Treatment and Delay

Descriptive statistics for identifying the quantity of numbers 1-10 can be seen in Table 15. For comparison, data in the column totals depict mean pre-test and post-test scores for Developmental Delay and Non-Developmental Delay groups collapsed across teaching methods. Similarly, row totals depict pre-test and post-test means for the two teaching methods collapsed across Developmental Delay/Non-Developmental Delay groups.

Table 15

Descriptive Statistics for Identifying the Quantity of Numbers (1-10) by Developmental Delay Groups and Treatment Group (N = 159)

		Traditional			Montessori SBS			Total		
DD		*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*	*n*
ND	Pre-Test	50.42	18.79	48	52.78	21.05	54	51.67	19.96	102
	Post-Test	61.67	19.28	48	63.70	20.40	54	62.75	19.81	102
DD	Pre-Test	29.23	24.65	26	42.58	28.52	31	36.49	27.42	57
	Post-Test	40.77	26.67	26	54.19	25.27	31	48.07	26.55	57
Total	Pre-Test	42.97	23.22	74	49.06	24.38	85	46.23	23.97	159
	Post-Test	54.32	24.16	74	60.24	22.62	85	57.48	23.47	159

Note. Pre- and post-test scores ranged from a minimum of zero to a maximum of 100. ND = non-developmental delay. DD = developmental delay.

Interaction between treatment and delay groups for identifying the quantity of numbers 1-10. Two 2 x 2 (Type of Intervention x Developmental Delay) factorial ANOVAs were conducted on the performance on identifying the quantity of numbers 1-10, separately for pre-test and post-test. As can be seen in Table 16, the ANOVAs did not reveal a significant interaction effect between the type of intervention and developmental delay groups for the quantity of numbers 1-10 on the pre-test, $F(1, 155) = 2.14$, $p = .146$, partial η^2 = .01, nor post-test, $F(1, 155) = 2.39$, $p = .124$, partial η^2 = .015. Because the interaction effect was not significant, the main effects are reported.

Table 16

Hypothesis #4: Two 2-Way Analyses of Variance of Pre- and Post-Test Identifying the Quantity of Numbers (1-10) Scores with Developmental Delay Groups and Type of Instruction as Fixed Factors

	Pre-Test				Post-Test			
Source	*df*	*MS*	*F*	*p*	*df*	*MS*	*F*	*p*
DD	1	8947.78	17.41	.010	1	8400.19	17.03	< .001
Type of Instruction	1	2242.50	4.36	.038	1	2171.80	4.40	.038
DD *Type of Instruction	1	1097.04	2.14	.146	1	1178.05	2.39	.124
Error	155	513.89			155	493.33		
Total	159				159			

Note. DD = developmental delay.

Main effects for identifying the quantity of numbers 1-10 for treatment. There was a significant main effect for type of instruction for identifying the quantity of numbers on the pre-test $F(1, 155) = 4.36$, $p = .038$ partial η^2 = .55, and post-test, $F(1, 155) = 4.40$, $p = .038$, partial η^2 = .55. The results suggest that students in the Montessori SBS group outperformed students in the Traditional group on the pre-test and post-test for identifying the quantity of numbers 1-10.

Main effects of identifying the quantity of numbers 1-10 for delay. ANOVAs revealed a significant main effect for Developmental Delay for the quantity of numbers 1-10 on the pre-test, $F(1, 155) = 17.41$, $p < .001$, partial $\eta^2 = .11$, and post-test, $F(1, 155) = 17.03$, $p < .001$, partial $\eta^2 = .98$. These results indicate that students identified as having a developmental delay performed more poorly than children without developmental delays on both pre- and post-tests for identifying the quantity of numbers 1-10.

Paired Samples *t* Tests to Assess Pre/Post Differences in Identifying the Quantity of Numbers 1-10 by Treatment and Delay

Four paired samples *t* tests were conducted to compare identifying the quantity of numbers 1-10 before and after the treatment for both treatment groups (Traditional vs. Montessori SBS) and developmental delay groups (Developmental Delay vs. Non-Developmental Delay) collapsed across groups.

Treatment groups/Quantity of numbers 1-10. Two paired samples *t* tests were conducted to compare performance on the quantity of numbers 1-10 before and after the intervention for treatment groups collapsed across developmental delay groups. There was a significant increase in identifying the quantity of numbers 1-10 for students in the Traditional group from the pre-test ($M = 42.97$, $SD = 23.22$) to the post-test ($M = 54.32$, $SD = 24.16$); $t(73) = -8.24$, $p < .001$. There was also a significant increase in identifying the quantity of numbers 1-10 for students in the Montessori SBS group on the post-test ($M = 60.24$, $SD = 22.62$), compared to the pre-test ($M = 49.06$, $SD = 24.38$), $t(73) = -46.54$, $p < .001$. These results suggest that both mathematical interventions improved performance on identifying the quantity of numbers 1-10. More specifically, these results suggest that when preschool children are provided Traditional or Montessori SBS lessons, learning to identify numbers 1-10 will improve. Results of the paired samples *t* tests can be seen below in Table 17.

Table 17

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Identifying the Quantity of Numbers (1-10) by Treatment Groups

Variable		*M*	*SD*	*SE*	95% Confidence Interval *LL*	95% Confidence Interval *UL*	*p*
Treatment							
	Traditional	-11.35	11.86	1.38	-14.10	-8.60	< .001
	Montessori SBS	-11.18	15.77	1.71	-14.58	-7.78	< .001

Note. LL = lower level. UL = upper level.

Delay groups/Quantity of numbers 1-10. Similarly, two paired samples *t* tests were conducted to compare performance on identifying the quantity of numbers 1-10 from pre- to post-test for developmental delay groups (Developmental Delay vs. Non-Developmental Delay) collapsed across groups. As seen in Table 28, there are statistically significant differences between identifying the quantity of numbers 1-10 on pre- and post-tests for students with and without developmental delays ($p < .001$). That is, there was a significant increase in performance on identifying the quantity of numbers 1-10 for students without developmental delays on the post-test ($M = 62.75$, $SD = 19.81$), compared to the pre-test ($M = 51.67$, $SD = 19.96$), $t(101)$, -8.58, $p < .001$. There was also a significant increase for students with developmental delays on the post-test ($M = 48.07$, *SD* = 26.55), compared to the pre-test (*M* = 36.49, *SD* = 27.42), $t(56) = -5.54$, $p < .001$. These results suggest that children with and without developmental delays, when presented with Traditional or Montessori SBS lessons, demonstrated an improvement in mathematical skills in identifying the quantity of numbers 1-10.

Table 18

Mean Difference Scores and Paired Samples t Test Results to Compare Pre/Post-Tests of Identifying the Quantity of Numbers (1-10) by Developmental Delay Groups

					95% Confidence Interval		
Variable		*M*	*SD*	*SE*	*LL*	*UL*	*p*
DD							
	No Delay	-11.08	13.04	1.29	-13.64	-8.52	< .001
	DD	-11.58	15.79	2.09	-15.77	-7.39	< .001

Note. LL = lower level. UL = upper level. DD = developmental delay.

Teacher Survey Responses

Reliability of Administering the Intervention

Descriptive statistics that reflected teacher's perceptions (*N* = 13) of the degree to which they reliably administered the interventions in their classroom during the study are presented in Table 19. More than half (53.9%) of the teachers of the Traditional method agreed that they taught the lessons for 10 minutes daily, compared to only approximately 8% of the teachers who taught the Montessori SBS intervention. These results reinforce the reliability of teachers in the Traditional group implementing the lessons on a daily basis during the study, but not the teachers in the Montessori SBS group.

Table 19
Teacher's Perceptions on the Reliability of Administering Traditional and Montessori SBS Interventions During the Study by Percentage (N = 13)

	Taught Lessons for 10 Minutes Daily				
Treatment	SD	D	N	A	SA
Traditional	0.0	7.7	0.0	23.1	30.8
Montessori SBS	0.0	7.7	23.1	0.0	7.7
Total	0.0	15.4	23.1	23.1	38.5
	Taught Lessons for More Than 10 Minutes Daily				
	SD	D	N	A	SA
Traditional	0.0	23.1	15.4	15.4	7.7
Montessori SBS	0.0	7.7	7.7	15.4	7.7
Total	0.0	30.8	23.1	30.8	15.4

Note. SD = strongly disagree. D = disagree. N = neutral. A = agree. SA = strongly agree.

Confidence in Teaching

Descriptive statistics in Table 20 reflect teacher's perceptions ($N = 13$) of the degree to which they had confidence in their teaching of the interventions in the classroom. Most of the thirteen teachers (77%) reported not struggling to teach the intervention, more teachers (84%) reported that they perceived that they had taught the interventions effectively in the classroom. Based on the results, more teachers (69.3%) felt, that in order to feel confident, they needed more practice in teaching the lessons before implementation. Overall, most teachers in this study (61.6%) felt confident in teaching the lessons during this study.

Table 20

Teacher's Perceptions of Self-Efficacy in Teaching the Intervention (N = 13)

	Effectiveness of Teaching the Lessons				
Treatment	SD	D	N	A	SA
Traditional	0.0	0.0	7.7	23.1	30.8
Montessori SBS	0.0	0.0	7.7	15.4	15.4
Total	0.0	0.0	15.4	38.5	46.2
	Struggled with Teaching the Lessons				
	SD	D	N	A	SA
Traditional	30.8	7.7	23.1	0.0	0.0
Montessori SBS	15.4	23.1	0.0	0.0	0.0
Total	46.2	30.8	23.1	0.0	0.0
	Need More Practice in Teaching Lessons Before Implementation				
	SD	D	N	A	SA
Traditional	23.1	30.8	7.7	0.0	0.0
Montessori SBS	0.0	15.4	7.7	15.4	0.0
Total	23.1	46.2	15.4	15.4	0.0
	Confidence in Teaching the Lessons				
	SD	D	N	A	SA
Traditional	0.0	7.7	7.7	23.1	23.1
Montessori SBS	0.0	0.0	23.1	15.4	0.0
Total	0.0	7.7	30.8	38.5	23.1

Note. SD = strongly disagree. D = disagree. N = neutral. A = agree. SA = strongly agree.

Willingness to Use Lessons After the Study

Table 21 reflects teachers' perceptions ($N = 13$) of their willingness to use either the Traditional or Montessori lessons in their classrooms after the study. That is, each teacher responded to the question in regard to the specific method that they had taught (Traditional vs. Montessori). All teachers felt the lessons in this study would be helpful to continue to use in their classrooms after the study. Most teachers (92.4%) reported that they plan to use these lessons in their classrooms in the future.

Table 21

Teacher's Perceptions on the Willingness to Use the Lessons After the Study (N = 13)

	Lessons Will Be Helpful in the Classroom After the Study				
	SD	D	N	A	SA
Traditional	0.0	0.0	0.0	23.1	38.5
Montessori SBS	0.0	0.0	0.0	15.4	23.1
Total	0.0	0.0	0.0	38.5	61.5
	Plan to Use Lessons in the Classroom After the Study				
	SD	D	N	A	SA
Traditional	0.0	0.0	7.7	23.1	30.8
Montessori SBS	0.0	0.0	0.0	23.1	15.4
Total	0.0	0.0	7.7	46.2	46.2

Note. SD = strongly disagree. D = disagree. N = neutral. A = agree. SA = strongly agree.

Student's Understanding of the Lessons Presented

Teachers' perceptions ($N = 13$) of student's understanding of the lessons presented during the study are shown below in Table 22. Approximately three-quarters of teachers (77%) felt that all students understood the lessons taught in this study. Most teachers (61.5%) reported that all students learned to count orally, while other teachers (23.1%) disagreed. However, all teachers in the Montessori group (38.5%) reported that students learned to count orally through the lessons presented in the study. Most teachers who responded to the survey (46.2%) agreed that students learned to understand one-to-one correspondence through the lessons in this study; however, results indicate that teachers did not perceive student's understanding of one-to-one correspondence based on methods used (Traditional vs. Montessori SBS).

Table 22

Teacher's Perceptions of Student's Level of Difficulty When Presented with Interventions During the Study (N = 13)

	All Students Understood the Lessons				
Treatment	SD	D	N	A	SA
Traditional	0.0	23.1	0.0	15.4	23.1
Montessori SBS	0.0	0.0	0.0	30.8	7.7
Total	0.0	23.1	0.0	46.2	30.8
	All Students Learned to Count Orally				
	SD	D	N	A	SA
Traditional	15.4	7.7	15.4	23.1	0.0
Montessori SBS	0.0	0.0	0.0	30.8	7.7
Total	15.4	7.7	15.4	53.8	7.7
	All Students Learned to Understand One-to-One Correspondence				
	SD	D	N	A	SA
Traditional	15.4	30.8	0.0	7.7	7.7
Montessori SBS	0.0	7.7	23.1	0.0	7.7
Total	15.4	38.5	23.1	7.7	15.4

Note. SD = strongly disagree. D = disagree. N = neutral. A = agree. SA = strongly agree.

Conclusion

Chapter IV presented the results of the study. Descriptive statistics were included as well as the results of data analyses for each hypothesis. Findings from student data and data from teacher surveys were also reported. Chapter V will explain the findings, limitations, and recommendations for further study.

CHAPTER V

DISCUSSION

This study investigated the relationship among the achievement of preschool students, with and without developmental delays, on their ability to count, identify, order, and identify the quantity of numbers 1-10 when taught using the Montessori SBS vs. Traditional methods. Interpretations of the findings are presented in the context of the current literature. Implications for practice, study limitations, and recommendations for future research conclude this chapter.

Interpretation of the Findings

Across the four skill sets tested in these preschool participants - ability to count, identify, order, and identify the quantity of numbers 1-10 - the findings of this study can be summed up in four main points. First, students with and without DD all improved following lessons. Second, students without DD performed better, compared to students with DD. Third, the degree of improvement was similar for students with and without DD. Fourth, the evidence suggests that improvement in both student groups was similar for Montessori SBS and traditional curricular methods.

According to the literature, lower performance of numeracy skills for preschool-aged children has the potential to be an indicator of developmental delays at an earlier age (Bassok et al., 2016; Cimpian et al., 2016; Dunphy et al., 2014; Nguyen et al, 2016). The findings in this study are consistent with the literature and suggest that students with developmental delays performed more poorly in counting, identifying, and identifying the quantity of numbers 1-10, compared to children without developmental delays.

The teacher survey in this study investigated teacher perceptions of the degree to which they implemented the lessons as scheduled, their effectiveness in delivering the lessons, and the likelihood that they would use the lessons from the intervention following the study. Findings from the teacher survey data are discussed in detail below.

Counting numbers 1-10. This study tested the hypothesis that the Montessori SBS instructional method in counting numbers 1-10 might be more effective than the traditional instructional method, as measured by the performance of preschool students with and without developmental delays. The data does not support the hypothesis. In addition to the finding of no difference between the Montessori SBS and Traditional treatments, the data also show that (a) students with developmental delays performed more poorly than children without developmental delays, (b) that all students in all groups improved from the pre-test to post-test, and (c) improvements from the pre-test to post-test were similar for students with and without developmental delays for counting numbers 1-10.

The literature suggests that counting is one of the earliest skills a child acquires (Carpenter et al., 2017; Kose & Arslan, 2015; Lefmann & Combs-Orme, 2013). In order to develop this skill, children must learn through repetition, observation, and a stimulating environment. When provided with evidence-based instruction and early intervention methods, these types of environments are evident and have the potential to serve as precursors for early prevention of learning difficulties in preschool (Gersten et al., 2015; Kyttala et al., 2015). Therefore, by including an effective instructional method or intervention, increases in children's knowledge of counting numbers may improve (Davenport & Johnson, 2015; DuPaul et al., 2015; Hinton et al., 2016; Khomais, 2014).

Moreover, research attributes a child's ability to count and verbally identify numbers on a child's instinctive knowledge and working memory (Feza, 2016; Kroesbergen et al., 2014; Kyttala et al., 2015; Pinhas et al., 2014). Instinctive knowledge and working memory are built through the use of hands-on manipulatives, modeling, repetition, and exploration of numbers (Gitter, 1967). The inclusion of hands-on materials is essential to developing counting skills as children make connections to tangible items (Brueggemann & Gable, 2018; Carpenter et al., 2017; Purpura & Lonigan, Voustina, 2016). More specifically, research shows that the inclusion of concrete and hands-on materials will make mathematical instruction more effective, and improvements, such as those seen in this study, will also be evident (Ball, 1992; Chisnall & Maher, 2007; Furner & Worrell, 2017; Laski et al., 2015; Steedly et al., 2008; Thompson, 1994; Uttal et al., 1997). Although the findings in this study reflect no differences between the two intervention methods, data indicate that both Montessori SBS and Traditional instruction improved counting skills of students with and without developmental delays. Thus, instruction and interventions that include reinforcement of instinctive knowledge, builds working memory, and incorporates hands-on materials, may be ideal for improving a child's knowledge in counting.

Similarly, research in early numeracy skills raises concerns about whether children with developmental delays in mathematics can be identified based on counting skills performance and early numeracy skill development. In this study, children with developmental delays performed more poorly than children without delays on their performance of counting 1-10. This result is consistent with the literature that children with developmental delays typically experience serious deficits in counting as early as three and four years old (Nguyen et al., 2016). It is relevant to reiterate that the children in this study who were identified as having a developmental delay were not pre-diagnosed by a certified medical professional. That is, it cannot be assumed that all of the children in the developmental delay group had an actual delay. Consequently, it cannot be determined that results of the children with and without developmental delays are entirely accurate, and that although differences between delay groups were identified, the uncertain identification of children with and without delays is not sufficient to support the literature.

Finally, Figure 6 reflects pre- and post-test performance on counting numbers 1-10 for both Montessori SBS and Traditional instruction groups, disaggregated by Delay versus No Delay groups for the purpose of showing that the degree of improvement for students with developmental delays was similar to students without developmental delays. This visual representation is consistent with the research that indicates that interventions, such as those used in this study, work to improve counting skills.

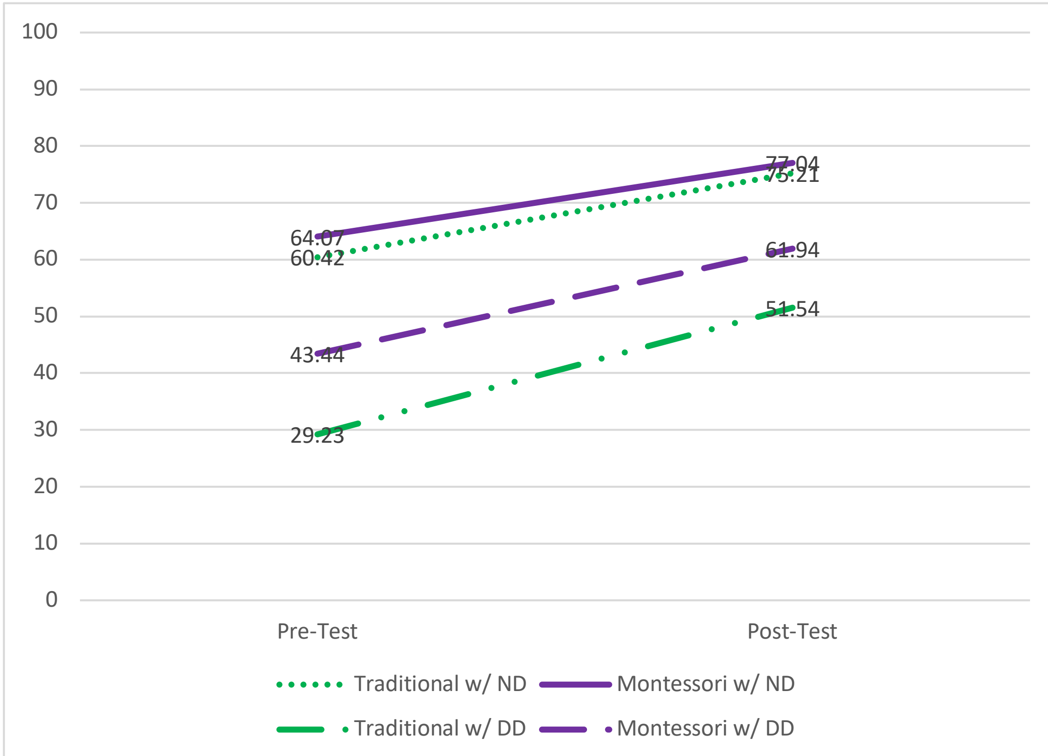


Figure 6. Means for counting numbers 1-10 from pre- to post-test for students with and without developmental delays using Montessori SBS vs. Traditional methods. ND = non-developmental delay; DD = developmental delay.

Identifying numbers 1-10. The second hypothesis investigated whether the Montessori instructional method in identifying numbers 1-10 might be more effective than the traditional instructional method, as measured by the performance of preschool students with and without developmental delays. Once again, the data did not support the hypothesis. Similar to the findings for Hypothesis 1, in addition to the finding of no difference between treatment groups on this skill, the data also show that (a) students with developmental delays performed more poorly than children without developmental delays, (b) all students in all groups improved from the pre-test to post-test, and (c) improvements from the pre-test to post-test were similar for students with and without developmental delays for identifying numbers 1-10.

Although students were randomly assigned to groups in this study, the scores of students who received the Montessori SBS instruction were higher for identifying numbers 1-10 at the start of the study, compared to students who received the Traditional instruction. Consequently, it is not possible to assume that any treatment group differences in student ability to identify numbers were because of the intervention. That is, because scores for the two groups were different prior to receiving the intervention, we must assume that there was some confound that was not solved by the randomization process. More specifically, these results are suggestive of unequal groups, which will be discussed further in this chapter as a limitation to this study.

Because the identification of numbers is one of the first numeracy skills to develop in children, this is one of the first skills that researchers have investigated to identify delays in mathematics at earlier ages (Gersten et al., 2005; Nguyen et al., 2016; Purpura et al., 2015). Without proper identification of numbers, children could be unsuccessful in learning other numeracy skills. The literature that suggests deficits in number identification predict other mathematical difficulties, suggests that finding that children with developmental delays performed more poorly, compared to children without developmental delays, suggests that these children may experience future deficits (Bassok et al., 2016; Cimpian et al., 2016; Hannula-Sormunen et al., 2015; Lee & Md-Yunus, 2016; Purpura et al., 2015; Reardon & Portilla, 2016; West, 2017). However, similar to the discussion of counting numbers 1-10, students were not medically diagnosed with a developmental delay prior to the study and therefore, this uncertainty may possibly skew the results of children with and without developmental delays. The assignment of children to either the developmental delay or no delay group was made on an individual level based upon practitioners' knowledge of the particular child; therefore, it is reasonable to assume that each child was appropriately assigned to either the delay or no delay group. The fact that the data also show the expected pattern of results for the delay and no delay groups, provides additional evidence that the findings related to this variable are valid. Consequently, it is not possible to assume that children with developmental delays performed more poorly than children without developmental delays.

Once children begin to develop counting skills, they achieve the ability to identify numbers (Kroesbergen, van't Noordende, & Kolkman, 2014; Kyttala et al., 2015; Pinhas, Donoahue, Woldorff, & Brannon, 2014). This is an important process that is used to name and count pictures, known as one-to-one correspondence (Carpenter et al., 2017; Hu et al., 2016; Kose & Aslan, 2015; Marmasse et al, 2000; Reedal, 2010). One-to-one correspondence can be difficult to achieve (Izard et al., 2014). Therefore, the use of instructional approaches and early interventions in numeracy skills are essential for preschoolers. Montessori SBS and Traditional instruction are both considered effective early intervention methods for preschoolers, which is consistent with the finding in this study that students showed a stronger understanding of identifying numbers 1-10 following either one of the interventions. This finding is consistent with the literature related to children's need for providing early interventions (Hinton et al., 2016; Kroesbergen et al., 2014; Kyttala et al., 2015) and structured activities that support their connections between counting and number structures (Voustina, 2016). In addition, these findings are consistent with the literature that indicates evidence-based instruction and interventions are needed to demonstrate improvements in identifying numbers for preschool children (Davenport & Johnson, 2015; DuPaul et al., 2015; Khomais, 2014; Passolunghi & Costa, 2016).

Consistent with the findings reported above for counting numbers 1-10, Figure 7 reflects pre- and post-test performance on identifying numbers 1-10 for both instruction groups and Delay/No Delay groups showing that, once again, similar improvements were evident for both sets of groups. That is, the degree of improvement in identifying numbers 1-10 for students with developmental delays was similar to students without developmental delays. This visual representation supports the research that interventions, such as those used in this study, work to improve identifying number skills 1-10.

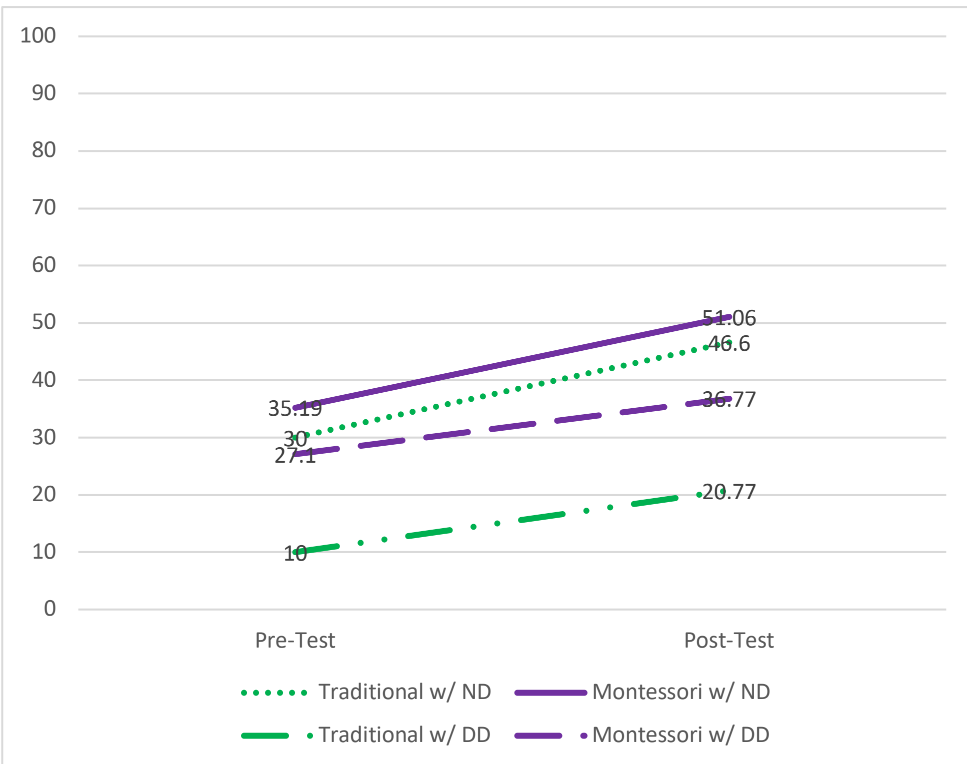


Figure 7. Means for identifying numbers 1-10 from pre- to post-test for students with and without developmental delays using Montessori SBS vs. Traditional methods. ND = non-developmental delay; DD = developmental delay.

Ordering numbers 1-10. The third hypothesis investigated whether the Montessori SBS instructional method in ordering numbers 1-10 might be more effective than the traditional instructional method, as measured by the performance of preschool students with and without developmental delays. Consistent with the previous findings, once again the data did not support the hypothesis. It is relevant to note that the interaction in this analysis was significant. However, the unusual pattern of results (i.e., no improvement) for students without developmental delays, along with the previously mentioned issue with unequal groups at pretest, does not permit an interpretation of the interaction. Additional findings related to this skill, similar to previous skills, include (a) students with developmental delays performed more poorly than students without developmental delays on the pre-test to post-test; (b) both treatment groups improved, but students without developmental delays did not improve; and (c) the degree of improvement for students with developmental delays was similar to students without developmental delays, with an exception for students without developmental delays in the traditional group.

Once children have established one-to-one correspondence, they can gain knowledge of cardinality; however, mastery of one-to-one correspondence is not required for learning the order of numbers. Therefore, when a child's counting improves, the child learns to recite the sequential numbers, place the numbers in a specific order, and use those numbers to name objects or items (Carpenter et al., 2017; Shusterman et al., 2016). This process is established and mastered using hands-on materials in an effective mathematics program. Although it appears that there was a treatment by time interaction for the students without developmental delays in the Traditional vs. Montessori SBS groups, it is unclear why the students without developmental delays taught with the traditional method showed little improvement between the pre- and post-tests.

Similar to the results of counting and identifying numbers 1-10, the findings for ordering numbers 1-10 also show that students with developmental delays performed more poorly than students without developmental delays on the pre-test to post-test. Although the research connects low performance of early mathematical skills to children with developmental delays, this study cannot conclude that the children identified with developmental delays were in fact children with deficits at the start of the study (Nguyen et al., 2016). Some students were placed into the Developmental Delay group because they were diagnosed with a developmental delay; however, the majority of the participants were grouped based on teacher, director, or researcher perceptions. Therefore, the difference in performance for ordering numbers 1-10 for students with and without developmental delays cannot contribute to the literature because of the uncertainty of identifying children with and without developmental delays. However, the consistent findings of deficits in performance for the delay group, compared to the no-delay group, suggest that the various methods used to identify the children with delays successfully classified the children into the right group.

Upon close inspection of Figure 8, with the exception of the unexplained diminished growth in the students with no developmental delay in the Montessori SBS group, the degree of improvement in ordering numbers 1-10 was similar for students with developmental delays compared to students without developmental delays in the Montessori SBS method. As the Montessori Method is structured around a well-ordered environment and didactic materials organized in a specific order, the emphasis on order and organization is prominent for mastering sequential skills in mathematics (Montessori, 1964). Manipulatives in Montessori are arranged in sequential order to introduce and master basic counting skills (Pickering, 1992). The results of this study highlight the impact of sequencing numbers for children and contributes to the literature that Montessori SBS instruction may be more effective for teaching the ordering of numbers than Traditional methods for children without developmental delays.

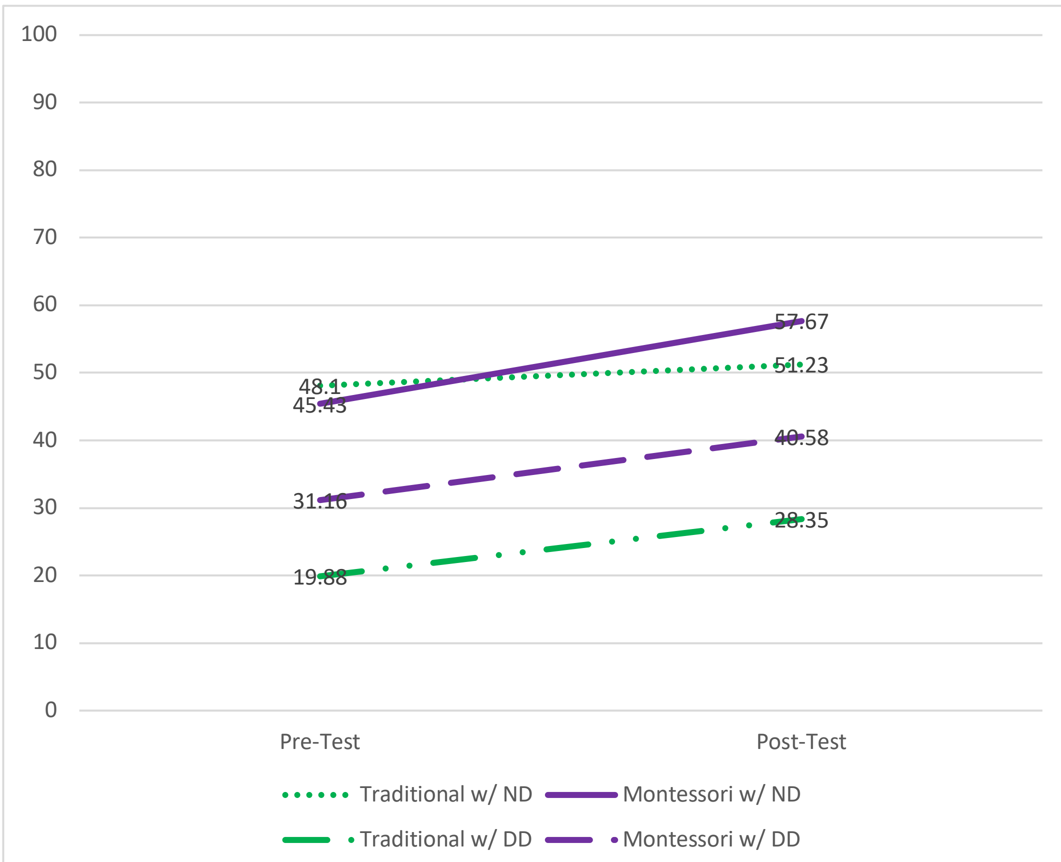


Figure 8. Means for ordering numbers 1-10 from pre- to post-test for students with and without developmental delays using Montessori SBS vs. Traditional methods. ND = non-developmental delay; DD = developmental delay.

Identifying the quantity of numbers 1-10. The fourth hypothesis in this study investigated whether there would be a difference in the performance of preschool students, with and without developmental delays, who were taught with Traditional or

Montessori SBS instructional methods in identifying the quantity of numbers 1-10. Once again, the data did not support the hypothesis. However, additional findings included (a) students with developmental delays performed more poorly than students without developmental delays on the pre-test to post-test, (b) all students in all groups improved from the pre-test to post-test, and (c) the degree of improvement for students with developmental delays was similar to students without developmental delays.

It was hypothesized that the Montessori SBS method would yield improved performance over the traditional method because the Montessori Method is aligned with a child's sensitive period, which refers to the child's readiness to experience and participate in learning activities (Montessori, 1964). However, this was not the case.

According to the literature, large gaps in developing early numeracy skills, particularly when identifying the quantity of numbers, are more noticeable for children with stronger math skills; therefore, it may be presumed that mathematical skills develop earlier than kindergarten (West, 2017). This study's findings identified a significant difference between students with and without developmental delays for identifying the quantity of numbers. Children without developmental delays outperformed their non-typically developing peers in both instructional groups. These differences in performance are consistent with what is in the literature regarding the studies related to academic achievement gaps in mathematics that possibly exist between children with developmental delays and their typically developing peers (Agrawal & Morin, 2016; Cimpian et al., 2016). However, these findings are similar to the results of how children performed on counting, identifying, and ordering numbers 1-10. Once again, it is relevant to note that, despite the fact that the criteria for establishing developmental delay in this study were variable across settings, the fact of diminished performance, compared to the group with no delay suggests that the criteria were successfully implemented.

Consequently, the results of this study show that all students with and without developmental delay in both Traditional and Montessori SBS groups improved from pre-test to post-test. These results suggest that both interventions worked to improve skills in identifying the quantity of numbers 1-10 for students with and without developmental delays. More specifically, the findings in this study are consistent with the literature indicating that when children are provided with an evidence-based intervention, improvements could be seen (Davenport & Johnson, 2015; DuPaul et al., 2015; Hinton et al., 2016; Khomais, 2014). In preparation for kindergarten, mathematical interventions related to number quantity are needed. This study incorporated two instructional approaches, Traditional and Montessori SBS, that may serve as mathematical interventions to improving a child's ability to identify the quantity of numbers. This study is also consistent with literature that discusses the importance of hands-on materials for mathematical instruction in identifying the quantity of numbers (Ball, 1992; Chisnall & Maher, 2007; Furner & Worrell, 2017; Laski et al., 2015; Steedly et al., 2008; Thompson, 1994; Uttal et al., 1997). When presented with a mathematical instruction that incorporates hands-on materials, such as Traditional or Montessori SBS, students increased their knowledge of identifying the quantity of numbers 1-10. Montessori materials, such as the varying colors in the Short Bead Stairs, were designed with 'quantity' in mind (Montessori, 1964). Many other mathematical counting materials, such as counting bears or blocks, were also designed for measuring quantity in early number skills. The findings in this study emphasize the significance of using hands-on materials that signify quantity in early childhood instruction and intervention.

Identifying the quantity of numbers is established when a child improves in other counting abilities (Shusterman et al., 2016). Once a child learns to count, they have the ability to generalize the quantity of larger numbers (Cheung et al., 2017). However, some children continue to struggle with complex mathematical numeracy concepts because of a limited foundation in the early development years (Mendizabal et al., 2015; Aunio et al., 2014). The results of this study emphasized the importance of early intervention instruction related to identifying the quantity of numbers for building strong foundational skills (Bashash et al., 2003; Bennett & Rule, 2005; Bouck et al., 2013; DuPaul et al., 2015; Gersten et al., 2005; Hudson et al., 2016; King et al., 2013). Children with and without developmental delays process learning quite differently; however, the results from this study suggest that all children in both intervention groups were able to learn and improve their skills in identifying the quantity of numbers 1-10. After visual inspection of the data, Figure 9 suggests that the degree of improvement for students with developmental delays was similar to students without developmental delays.

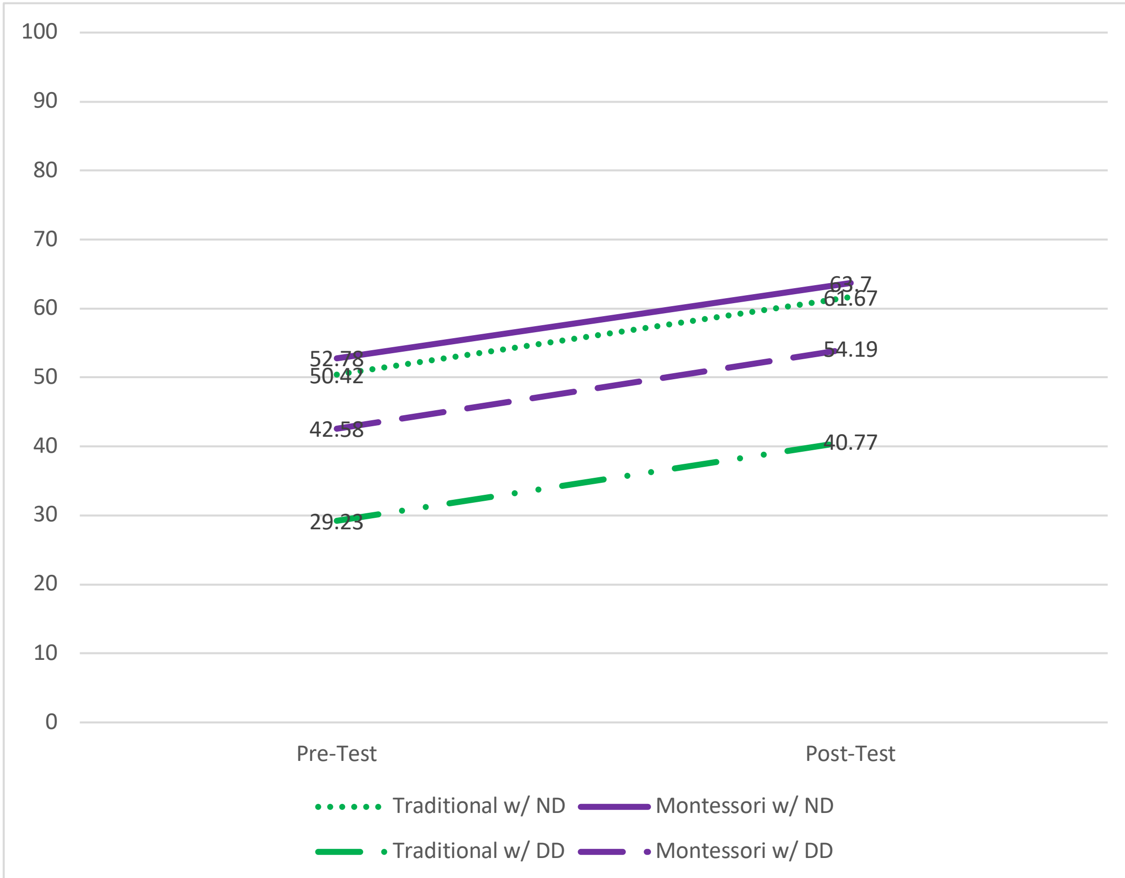


Figure 9. Means for identifying the quantity of numbers 1-10 from pre- to post-test for students with and without developmental delays using Montessori SBS vs. Traditional methods. ND = non-developmental delay; DD = developmental delay.

Other observational findings. Increases in mean scores between Traditional and Montessori SBS instructional groups were similar in this study. Although this demonstrates positive growth for students with and without developmental delays, it questions whether the length of the study was sufficient to identify significant differences between the groups. According to Dr. Maria Montessori (1967), children have an absorbent mind that allows them to unconsciously learn by interacting with the environment during early stages of development. In comparison to previous literature, this finding may be indicative that students in both groups were not presented with the material for a long enough period of time to identify differences between the instructional methods. This limitation will be discussed later in the chapter.

As the results from different counting skills were high for both, students with and without developmental delays, results were indicative that knowledge in identifying numbers and one-to-one correspondence could be present as well. However, these findings demonstrate that across all numeracy skill areas, students with and without developmental delays are weakest in identifying numbers 1-10 as seen in Figure 8 below. These differences show the level of difficulty for identifying numbers in early numeracy skills when compared to counting, ordering, and quantity of numbers. When considering skill development, these findings suggest that identifying numbers may be one of the last early numeracy skills developed for young children.

The order of learning the sequence of numbers is different for every child (Carpenter et al., 2017). Consequently, the results in this study emphasized that children with developmental delays may be less successful in determining whether a number is presented in sequence (Bashash et al., 2003). The findings may also suggest that because children learn at different rates, children with developmental delays may not have been ready to learn these skills, specifically the ability to order numbers. This skill may take longer to develop.

It was also noted that students without a developmental delay began the pre-test with a higher average than students with a developmental delay. These differences at the baseline assessment contribute to the research that children with developmental delays perform lower than typically developing peers (Nguyen et al., 2016). Whether this is a result of a lack of exposure to numeracy skills at an early age, or if these differences are attributed to the child's delay in development, a difference between students with developmental delays and those without was evident in all numeracy skill areas for this study. Ironically, there were also differences between students with developmental delays in both instructional groups. After further discussion with the director and teachers of each school, two of the three schools acknowledged that students with developmental delays were grouped by ability which is indicative of the difference in pretest scores for students with developmental delays. It can be speculated that groups assigned to the traditional intervention consisted of low ability groups. This speculation could explain the differences between the intervention groups.

The results also identify that students in the Montessori group began the study with a higher average than students in the Traditional group for both developmental and non-developmental delay groups. Unknowingly, most students in this study were grouped by ability; this was revealed at the end of the study through discussion held with the directors of two of the three schools that participated in the study. Although groups were randomly assigned, this separation of classes by ability allotted for a difference between Traditional and Montessori groups at the start of the intervention. This inequality at the pre-test will be discussed later in this chapter as this is a limitation to the study.

Based on the theories of Montessori (1949) and Piaget (1942), when children are ready to learn new skills, the child will master those skills in their environment. It is possible that students in this study were not provided enough time to cause a difference, or students were not ready to learn the skills presented. This study commenced at the beginning of the school year where many of the students in the study were either three years old or never exposed to prior mathematics instruction; hence, demonstrating how students may not have been prepared to learn these skills. Furthermore, the Montessori Method has an inclusive environment in which children in the Montessori setting are exposed to the methods throughout the day, unlike the students who were presented with the materials for only ten minutes daily in this study. The learning in a Montessori setting is not limited to the didactic materials but includes the process and the environment that surrounds the child (Montessori, 1967). This could explain the lack of findings for treatment, although the problems of unequal groups must also be considered.

Post hoc tests, such as Bonferroni, reportedly have the potential to reduce the power of the study and increase errors across groups (Nakagawa, 2004; Perneger, 2008). Therefore, post hoc tests were not conducted. Instead, effect sizes, exact *p* values, and confidence intervals were reported.

Teacher survey findings. Findings for reliability, confidence, willingness to use the lessons after the study, and perceptions of student's understanding of lessons presented from teacher participants in this study are described below.

Reliability. Preschool teachers generally agree that they are not consistent with teaching lessons and are ineffective in their instruction (Takunyaci & Takunyaci, 2014). In comparison to the literature, most teacher participants in this study reported that they taught their lessons each day for the minimum amount of time; however, there were still a number of teachers who did not. It is important that teachers are consistent and reliable in their teaching practices to ensure children are receiving the appropriate amount of practice for each skill. In this study, although most teachers reported they were consistent and reliable, concerns about the teachers who did not report they taught their lessons for the required daily time are raised. More specifically identified was the concern that more teachers in the Traditional group agreed that they taught their lessons for the minimum amount of time, while a small portion of teachers in the Montessori SBS group did not. This difference could either be a result of teachers not fully paying attention when responding to the survey questions, or teachers in the Montessori group not fully understanding the importance of teaching lessons daily. Either way, this result questions whether results of students' performance in the Montessori SBS group were entirely valid based on the lack of reliability from the Montessori SBS intervention teachers. These concerns set further limitations to this study and are discussed later in this chapter.

Confidence. Research reports a positive correlation between teacher beliefs and practices (Stipek et al., 2001). Teachers with a lower self-confidence in mathematics often enjoy mathematics less than others who do not believe themselves to have stronger skills for teaching mathematics. In comparison to the literature, teachers in this study reported that they did not struggle to teach the lessons, and that the lessons were taught effectively. Effectiveness of teaching lessons is often attributed to a teacher's adequate preparedness (Madu, 2016). Although specialized training was provided prior to the study, teachers felt that more practice and training in the lessons was needed.

Willingness to use after the study. Instead of attributing a child's understanding to a particular mathematical problem, some teachers focus on the child's skill level and ability (Stipek et al., 2001). Teachers who identify an intervention that helps children improve their skills are more willing to use the intervention in the classroom time and time again. Findings from this study suggest teachers found the interventions helpful and they plan to use the interventions for future instruction.

Children's understanding of lessons presented. This study's findings demonstrate that some teachers perceived their students to understand the lessons presented, while others did not. This finding may be indicative of how teachers' modeling and self-confidence can influence children's beliefs (Stipek et al., 2001). Children are receptive to their surroundings and although some children may understand some lessons, if teachers do not model appropriately or lack mathematical confidence, improvements in numeracy skills may not be evident. Only a small number of teachers in this study reported that most of their students learned to count. More specifically, when comparing the teachers in the two groups, all teachers in the Montessori SBS intervention group agreed that students in their classes learned to count; whereas, only some teachers in the Traditional group who reported that most students learned to count. Although the majority of teachers did not agree that their students understood counting by the end of the study, the teachers in the Montessori SBS group felt more strongly about their students' performance than teachers in the Traditional group. This difference in student understanding could be related to teacher self-confidence and instructional modeling. The study's findings also show that teachers in the Traditional group perceive their students learned one-to-one correspondence more than teachers in the Montessori SBS intervention group. Teacher perceptions were not indicative of actual participant findings but did provide insight into viewpoints of teachers from each instructional group.

Implications for Practice

This study attempted to identify relationships between preschool-aged students', with and without developmental delays, and their performance of counting, identifying, ordering, and identifying the quantity of numbers 1-10 using the Montessori SBS and

Traditional methods. However, this study did not support the hypotheses presented. Although no significant findings were identified, the discussion of the findings suggests that more focus should be tailored to the (1) screening and identification of children with developmental delays, (2) professional development, and (3) early childhood interventions.

Identification of children with developmental delays. Although the findings related to differences between children with and without developmental delays in this study were uncertain, there is still plausible implications for practice. As most of the children with developmental delays in this study were identified by adult perceptions and not a medical diagnosis, the results that students without developmental delays outperformed their peers with developmental delays on counting, identifying, ordering, and the quantity of numbers 1-10, is possibly skewed. However, this difference between the groups may contribute to the understanding that deficits in early numeracy skills, such as counting, identifying, ordering, and identifying the quantity of numbers, may be early identifiers of learning difficulties (Bassok et al., 2016; Cimpian et al., 2016; Dunphy et al., 2014; Gersten et al., 2015; Kyttlls et al., 2015). The results of this study, although undefined, may be indicative of deficits in mathematics at an early age.

Similarly, although delays in learning are deficits, lower performance does not always correlate to a developmental delay. Therefore, these deficits at an early age may not solely be indicative of a developmental delay; however, early identification of deficits or delays in mathematics have the potential to close learning gaps and serve as a screening tool for identifying children with developmental delays.

Professional development. The literature suggests that the use of manipulatives and interventions are essential for improving counting skills, knowledge, and understanding of number concepts for children with and without developmental delays (Agrawal & Morin, 2016; Bashash et al., 2003; D'Angelo & Iliev, 2012; Hewitt, 2001; Hudson et al., 2016; Peterson & McNeil, 2013; Post, 1981; Rosli et al., 2015). Although the results of the study did not determine if the use of Montessori SBS was better than traditional, or vice versa, the findings of this study did suggest that students with and without developmental delays improved in counting, identifying, and identifying the quantity of numbers for both Traditional and Montessori SBS instructional methods. Both of these instructional methods include the integration of hands-on materials for learning.

Furthermore, it might be expected that additional practice and professional development of differing mathematics instruction could lead to an increase in student growth. If teachers are equipped with the appropriate skills and teaching methods for instruction, then they may be more prepared to implement mathematics instruction more effectively. Similarly, teachers who feel inadequate in teaching certain foundational skills might be more inclined to seek professional development that supports these efforts.

Early childhood interventions. Additionally, these findings have the potential to inform programming of mathematical interventions in early childhood. Although the findings in this study were not conclusive about significant differences between instructional interventions, the findings in this study showed that when students were presented with an instructional method that included daily instruction and hands-on materials, learning of specific numeracy skills improved. Students in all groups were provided instruction that included visual representations, hands-on experiences, and repetition of concepts. Specifically, the findings from this study, although insignificant, may continue to inform future discussion regarding the reinforcement of instinctive knowledge, building working memory, and incorporating hands-on materials for improving a child's knowledge in numeracy skill development. Early childhood mathematics programs that offer hands-on experiences in conjunction with daily mathematics instruction, and structured activities that support connections between counting and number structures, have the potential to improve overall numeracy skills knowledge for preschool-aged children (Furner & Worrell, 2017; Laski et al., 2015; Voustina, 2016).

Limitations and Future Research

Despite the strengths of this study's quantitative approach with randomized groupings, some limitations should be mentioned. For example, limitations that include duration of the study, inequality of groups, instructional environment, and teacher survey will be discussed, along with recommendations for future research.

First, the length of the study served as a limitation. Due to time constraints, this study was conducted within a shortened window of opportunity that included three weeks of intervention and would be more credible if given more time to allow significant growth across groups. Although previous research suggested three weeks would be sufficient for identifying growth in early numeracy skills, it is possible to assume that children at a younger age and without possible developmental delays should be afforded approximately 9-12 weeks with a minimum of 30 minutes to develop mastery of skills through scaffolded experiences. Therefore, future studies should include more time for the study to allow for growth in different groups.

One of the most noticeable differences in the results of this study was a difference in the performance of groups at the start of the study. The results showed that scores for the students in the Traditional groups were lower than the Montessori SBS intervention group at pre-test. These results are indicative of a limitation to the study. However, after further discussion with the school directors and teachers, the results of this study can be better explained by a discrepancy between ability groups. It was later recognized that most students at two of the three schools, although groups were randomly assigned, were not randomly assigned within classes. Students were assigned to classes in each school based on ability and age levels. This was, unfortunately, not disclosed until after the completion of the study. Because students were grouped by ability before the study began, two discrepancies for students with developmental delays were identified. First, higher performing students with developmental delays were placed in the same class, and lower performing students in a separate class with lower performing peers. Secondly, it was not specifically identified if a student had a cognitive developmental delay or physical delay and possibly skewed the results of the developmental groupings. Thirdly, children in the traditional group were mostly three years of age, whereas children in the Montessori SBS intervention group were mostly four years of age. The differences between age groups and cognitive abilities at the different ages identify discrepancies between the treatment groups. These disparities are indicative of unequal groups within the study. Because students in the Montessori SBS and Traditional groups were not equal at the start of the study, the results of this study are not strong. In order to strengthen this study, future research should first ensure that classes within the study are equal before proceeding. More credibility could be given to this study if homogenous grouping was identified beforehand or if selective classes with homogenous grouping were incorporated.

Because the environment for students receiving the Montessori SBS instruction was outside of a Montessori program or classroom, further research in a Montessori infused classroom would be appropriate. The limited time of receiving Montessori instruction daily in mathematics may not solely indicate a student's performance as Montessori methods involve an ordered process, sequential reasoning, and a child's environment. It is also important to recognize that the classrooms in this study did not consist of a traditional multi-age Montessori setting in a Montessori school. Additionally, the interventions did not include a non-interrupted three-hour learning timeframe which is ideal in the Montessori classroom setting. More importantly, children in a Montessori environment are introduced to other didactic materials related to early numeracy skills (Spindle Box, Pink Tower, etc.) before exposure to the Montessori Short Bead Stairs; however, children in this study were not exposed to these materials. Lastly, although the researcher has experience observing, learning, and teaching in a Montessori classroom, she is not certified in Montessori. Thus, the teachers were trained by a non-Montessorian. Therefore, based on the limitations of the Montessori environment in this study, the findings are not indicative of a traditional Montessori classroom environment. The findings in the study are limited to the exposure to the individual lessons presented using the Montessori Short Bead Stairs. Future research could include a comparison between children in a Montessori setting and children in a traditional setting over a longer period of time.

Another limitation to the study was related to the teacher survey. Based on the results of the survey, some teachers reported that they did not teach their lessons for the required daily time of ten minutes. This raises questions about the study's implementation and reliability. From frequent visits and discussions with the teachers, it was noted that the teachers were meeting the minimum requirements of ten minutes of daily instruction; however, after receiving the results of the teacher survey, some teachers contradicted their statements. When considering future research, more preparations for teachers could be conducted and follow-up procedures implemented. A one-week training is recommended for teachers to practice and learn the instructional methods.

Based on the collection of data, this study lacked two important components about the participating teachers. First, a total of sixteen teachers participated in the implementation of the interventions; however, only thirteen completed the survey. It was questionable as to why some teachers did not participate in the survey. Three possible explanations for lack of participation of the three teachers could be that the teachers either did not receive the email sent, they did not make it a priority to submit their results, or they did not have access to a computer to be able to report their findings. Based on the fact that computers were not visible at any of the school locations during the study, the teachers in the study possibly did not have access to a computer. Regardless of which of the three possible explanations for not participating in the survey, future studies should take into account that preschool teachers are not always equipped with access to a computer. Therefore, it could have been more effective to allow teachers to complete a handwritten survey in lieu of an online questionnaire.

Secondly, the survey did not request demographic information about the teacher, such as ethnicity, age group the teacher is currently teaching, gender, years of service, and educational experience. These results could have been helpful in comparing teacher responses for each question, as well as comparing the information to overall student performance. A more in-depth survey that includes these demographic questions should be considered for future research.

Conclusion

Early numeracy skills are needed to support a strong foundation in mathematics and close learning gaps (Agrawal & Morin, 2016, Carpenter et al., 2017, Hannula-Sormunen at al., 2015). For mathematics development to be successful, interventions that include hands-on manipulatives are vital. Research concludes that when children are presented with manipulatives for mathematical instruction, improvement in the areas of numeracy skills are identified (Huntley-Fenner, 2001; Zhu et al., 2017). Similarly, a lack of early mathematical skills has the potential to predict later deficits in mathematics between children with and without developmental delays (Nguyen, 2016; Reardon & Portilla, 2016; Reid & Andrews, 2016).

However, the results of this study are unable to support the literature for several reasons. First, there were no differences between the Montessori SBS intervention and Traditional groups to identify if one intervention was better than the other. Similarly, treatment groups were noticeably different at the pre-test, and therefore, it is uncertain as to whether the Montessori SBS intervention made a difference in the results. Consequently, the unclear diagnoses of children with developmental delays does not support the results that students without developmental delays outperformed their peers with developmental delays. Although improvements for students with and without developmental delays were observed in the results, the inconsistent groupings made it impossible to identify statistical differences.

As children with developmental delays are not often identified until after the age of seven, it is important to provide early interventions in mathematics to facilitate building a strong foundation for all learners. It is also important that teachers of all grade levels understand the importance of early interventions. Although findings in this study are not significant, the results show that the use of Traditional or Montessori SBS instruction could serve as an early intervention in mathematics as improvements in the performance of counting, identifying, and identifying the quantity of numbers 1-10 were discovered. These early intervention techniques have the potential to extend the current understanding that appropriate evidence-based mathematical interventions for preschool-aged children are needed to develop early numeracy skills. However, further investigation of equal groups may clarify the relationships between Traditional and Montessori instruction on children with and without developmental delays for each early numeracy skill.

References

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research and Practice, 31*(1), 34-44. doi:10.1111/ldrp.12093

Alvarado, M. (2015). The utility of written numerals for preschool children when solving additive problems. *Estudios de Psicologia, 36*(1), 92-112. doi:10.1080/02109395.2014.1000026

American Psychiatric Association. (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.). Arlington, VA: American Psychiatric Publishing.

Anderson, C., & Palm, T. (2017). Characteristics of improved formative assessment practice. *Education Inquiry, 8*(2). https://doi.org/10.1080/20004508.2016.1275185

Ansari, A., & Winsler, A. (2016). Kindergarten readiness for low-income and ethnically diverse children attending publicly funded preschool programs in Miami. *Early Childhood Research Quarterly, 4*(37), 69-80. doi:10.1016/j.ecresq.2016.06.002

Antell, S., & Keating, D. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695-701. doi:10.2307/1130057

Aunio, P., Korhonen, J., Bashash, L., & Khoshbakht, F. (2014). Children’s early numeracy in Finland and Iran. *International Journal of Early Years Education, 22*(4), 423-440. doi:10.1080/09669760.2014.988208

Ball, D. K. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator, 16*, 14-18. Retrieved from https://www.aft.org/sites/default/files/ periodicals/ae_summer1992_ball.pdf

Barbieuri, I. T. (2016). The role of the educator in a Montessori classroom. *Romanian Journal for Multidimensional Education, 8*(1), 107-123. doi:10.18662/rrem/2016.0801.07

Barnett, W. S., Carolan, M. E., Squires, J. H., Brown, K., & Horowitz, M. (2015). The State of Preschool 2014: State preschool yearbook. New Brunswick, New Jersey: National Institute for Early Education Research.

Bashash, L., Outhred, L., & Bochner, S. (2003). Counting skills and number concepts of students with moderate intellectual disabilities. *International Journal of Disability, 50*(3), 325345. doi:10.1080/1034912032000120480

Bassok, D., Finch, J. E., Lee., R., Reardon, S. F., & Waldfogel, J. (2016). Socioeconomic gaps in early childhood experiences: 1998-2010. *AERA Open, 2*(3), 1-22. doi:10.1177/2332858416653924

Bassok, D., & Latham, S. (2016). Kids today: Changes in school-readiness in an early childhood era. *Ed Policy Works.* Retrieved from https://curry.virginia.edu/uploads/ resourceLibrary/35_Kids_Today.pdf

Bauch, J., & Hsu, H. (1988). Montessori: Right or wrong about number concepts? *Arithmetic Teacher, 35*(6), 8-11. Retrieved from https://eric.ed.gov/?q=Montessori %3a+ Right+or+Wrong+about+Number+Concepts%3f&pr=on&id=EJ366591

Bennett, P., & Rule, A.C. (2005). Hands-on long division with skittles for students with learning disabilities. *Teaching Exceptional Children Plus, 1*(5). Retrieved from http://files.eric.ed.gov/fulltext/EJ966522.pdf

Bennett, P., Elliott, M., & Peters, D. (2005). Classroom and family effects on children's social and behavioral problems. *The Elementary School Journal, 105*(5). 461-480. doi:10.1086/431887

Bouck, E., Joshi, G., & Johnson, L. (2013). Examining calculator use among students with and without disabilities educated with different mathematical curricula. *Educational Studies in Mathematics, 83*(3), 369-385. doi:10.1007/s10649-012-9461-3

Boyle, C.A., Boulet, S., Schieve, L.A., Choen, R.A., Blumberg, S.J., Yeargin-Allsopp, M., ….Kogan, M.D. (2011). Trends in the prevalence of developmental disabilities in US children, 1997-2008. *American Academy of Pediatric.* Online submission. doi:10.1542/peds.2010-2989

Brendefur, J., Strother, S., Thiede, K., Lange, C., & Surges-Prokop, M. (2013). A professional development program to improve math skills among preschool children in head start. *Early Childhood Education Journal, 41*(3), 187-195. doi:10.1007/s10643-012-0543-8

Brewer, J. & Daane, C. J. (2002). Translating constructivist theory into practice in primary-grade mathematics. *Education, 123*(2), 416-426. Retrieved from https://www.questia.com/ library/journal/1G1-98248746/translating-constructivist-theory-into-practice-in

Brooks, J. G., & Brooks, M. G. (1999). *In search of understanding: The case for constructivist classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.

Brown, K., & Lewis, C. W. (2017) A comparison of reading and math achievement for African American third grade students in Montessori and other magnet schools. *Journal of Negro Education, 86*(4), 439-448 doi:10.7709/jnegroeducation.86.4.0439

Brueggemann, A., & Gable, S. (2018). Preschoolers' selective sustained attention and early numeracy skills and knowledge. *Journal of Experimental Child Psychology, 171*(2), 138-147. doi:10.1016/j.jecp.2018.02.001

Buhaglar, M. A., & Murphy, R. (2008). Teachers' assessments of students' learning of mathematics. *Assessment in Education: Principles, Policy, & Practice, 15*(2). https://doi.org/10.1080/09695940802164192

Bulder, A., Omeroglu, E. (2018). An examination of the relationship between pre-school children's and their teachers' attitudes and awareness towards the environment. *Journal of Education and Learning, 7*(2), 221-229. doi:10.559/jel.v7n2p221

Caprioara, D., & Anghelide, M. (2016). Constructivist paradigm in the learning of school mathematics. *Bulletin of the Transylvania University of Brasov, Special Issue Series VII, 9*(51), 1-2016. Retrieved from https://web-b-ebscohost.com.ezproxy.barry.edu /ehost/pdfviewer/pdfviewer?vid=3&sid=fb895a30-b282-4138-8113-32d979e13322%40pdc-v-sessmgr01

Cakiroglu, U., & Taskin, N. (2016). Teaching numbers to preschool students with interactive multimedia: An experimental study. *Cukurva University Faculty of Education Journal, 45*(2), 1-22. Retrieved from https://www.researchgate.net/publication/312029950

Carbonneau, K. J., Marley, S. C., Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400. doi:10.1037/a0031084

Carpenter, T. P., Franke, M. L., Johnson, N. C., Turrou, A. C., & Wager, A. A. (2017). *Young children's mathematics: Cognitively guided instruction in early childhood education*. Portsmouth, New Hampshire: Heinemann.

Carr, M. & Lee, W. (2012). Learning stories: Constructing earner identities in early education. London, England: Sage Publications Ltd.

Chapin, S. H., & Eastman, K. E. (1996). Implementing the professional standards for teaching mathematics: External and internal characteristics of learning environments. *Mathematics Teacher, 89*(2), 112-15. Retrieved from https://eric.ed.gov/?id=EJ518933

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology, 92*(1), 22-36. doi:10.1016/j.cogpsych.2016.11.002

Chi, F. W., van Marie, K. & Geary, D. C. (2016). Predicting children's reading and mathematics achievement from early quantitative knowledge and domain-general cognitive abilities. *Frontiers in Psychology, 7*, 775. doi:10.3389/fspsyg.2016.00775

Chisnall, N., & Maher, M. (2007). Montessori mathematics in early childhood education. *Curriculum Matters,* 3. *Academic OneFile.* Retrieved from http://www.nzcer.org.nz/default.php?cPath=139_134_290

Chumark, C., & Puncreobutr, V. (2016). Developing basic mathematical skills of preschool children by using plasticized clay. *Journal of Education and Practice, 7*(12), 180-183. Retrieved from https://files.eric.ed.gov/fulltext/EJ1099541.pdf

Cimpian, J. R., Lubienski, S. T., Timmer, J. D., Makowski, M. B., and Miller, E. K. (2016). Gender gaps in math closed: Achievement, teacher perceptions, and learning behaviors across two ECLS-K cohorts. *AERA Open, 2*(4), 1-19. doi:10.1177/2332858416673617

Clements, D., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York, NY: Routledge of the Taylor & Francis Group.

Corcoran, L., & Steinley, K. (2017). Early childhood program participation, results from the national household education surveys program of 2016. *National Center for Education Statistics.* Retrieved from https://nces.ed.gov/pubs2017/2017101.pdf

Damnjanovic, Z. (2012). Truth through proof: A formalist foundation for mathematics by Alan Weir. *Book Review, 72*(2), 415-418. https://doi.org/10.1093/analys/ans011

D'Angelo, F. & Iliev, N. (2012). Teaching mathematics to young children through the use of concrete and virtual manipulatives. *Online Submission.* Retrieved from https://eric.ed.gov/?id=ED534228

Davis, P., & Hersh, R. (1981). *The mathematical experience.* Boston, MA: Birkhauser.

Davenport, L. A., & Johnston, S. S. (2015). Using most-to-least prompting and contingent consequences to teach numeracy in inclusive early childhood classrooms. *Topics in Early Childhood Special Education, 34*(4), 250-261. doi:10.1177/0271121413518824

Desoete, A., Praet, M., Van de Velde, C., Craene, B., & Hantson, E. (2016). Enhancing mathematical skills through interventions with virtual manipulatives. *International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives,* 171-187. doi:10.1007.978-3-319-32718-1_8

Devine, A., Soltész, F., Nobes, A., Goswani, U., & Szu ‘cs, D. (2013). Gender differences in developmental dyscalculia depend on diagnostic 441 criteria. *Learning and Instruction, 27,* 31–39. http://dx.doi.org/10.1016/j.learningstruc.2013.02.004

Devine, A., Hill, F., Carey, E., & Szu’cs, D. (2018). Cognitive and emotional math problems largely dissociate: Prevalence of developmental dyscalculia and mathematics. *Journal of Education Psychology, 110*(3), 431-444. http://dx.doi.org/10.1037/edu0000222

Digest of Education Statistics. (2016). Children 3 to 21 years old served under Individuals with Disabilities Act, part B, by age group and sec, race/ethnicity, and type of disability: 2014-15. *National Center for Education Statistics.* Retrieved from https://nces.ed.gov/ programs/digest/d16/tables/dt16_204.50.asp

Dolschield, S., Winter, C., Ostrowski, L., & Penke, M. (2017). The many ways quantifiers count: Children’s quantifier comprehension and cardinal number knowledge are not exclusively related. *Cognitive Development, 44,* (1), 21-31. doi:10.1016/j.cogdev.2017.08.004

Dunphy, E., Dolley, T., & Shield, G. (2014). Mathematics in early childhood and primary education (3-8 years). *National Council for Curriculum and Assessment, Research Report No. 17*. Retrieved from https://www.ncca.ie/media/1494/ maths_in_ecp_education_ theories_progression_research/report_17.pdf

DuPaul, G. J., Kern, L., Caskie, G. I., Volpe, R. J., & Gilman, R. (2015). Early intervention for young children with attention deficit hyperactivity disorder: Prediction of academic and behavioral outcomes. *School Psychology Review, 44*(1), 3-20. doi:10.17105/SPR44-1.3-20

Education for All Handicapped Children's Act of 1975, Pub. L. 94-142, 89 Stat. 773. Retrieved from https://www.govtrack.us/congress/bills/94/s6/summary #libraryofcongress

Emenaker, C. (1996). A problem-solving based mathematics course and elementary teacher's beliefs. *School Science and Mathematics, 96*(2). https://doi.org/10.1111/j.1949-8594.1996.tb15814.x

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object arrays. *Cognition, 97*(3), 295-313. doi:10.1016/j.cognition.2004.09.010

Feza, N. (2016). Teaching 5- and 6-year-olds to count: Knowledge of south African educators. *Early Childhood Education Journal, 44*(5), 483-489. doi:10.1007/s10643-015-0736-z

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education, 31*(3), 195-207. doi:10.1177/0741932508327467

Florida Department of Education. (2014). *Mathematics Florida standards (MAFS): Kindergarten.* Retrieved from http://www.fldoe.org/core/fileparse.php/12087/urlt/GK_Mathematics_Florida_Standards.pdf

Florida Early Learning Coalition. (2011). *Florida early learning and developmental standards for four-year olds.* Retrieved from http://flb+5.floridaearlylearning.com/BT5_Uploads/feldsfyo.pdf

Foreman, J. L., & Gubbins, E. J. (2015). Teachers see what ability scores cannot: Predicting student performance with challenging mathematics. *Journal of Advanced Academics, 26*(1), 5-23. doi:10.1177/1932202X14552279

Foegen, A., Jiban, C., & Deno, S. (2007). Progress monitoring measures in mathematics: A review of the literature. *The Journal of Special Education, 41*(2), 121-139. Retrieved from https://files.eric.ed.gov/fulltext/EJ775117.pdf

Frierson, P. R. (2014). Maria Montessori's epistemology. *British Journal for the History of Philosophy, 22*(4), 767-791. doi:10.1080/09608788.2014.960794

Frierson, P. R. (2016). Making room for children's autonomy: Maria Montessori's case for seeing children's incapacity for autonomy as an external failing. *Journal of Philosophy of Education, 50*(3), 332-350. https://doi.org/10.1111/1467-9752-12134

Furner, J. M., & Worrell, N. L. (2017). The importance of using manipulatives in teaching math today. *Nova Southeastern University: Transformations, 3*(1), 1-25. Retrieved from https://nsuworks.nova.edu/transformations/vol3/iss1/2

Fuson, K. C., Clements, D. H., & Sarama, J. (2015). Making early math education work for all children. *Phi Delta Kappan, 97*(3), 63-68. doi:10.1177/0031721715614831

Gallistal, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4,*59-65. Retrieved from https://www.academica.edu/32410391/Nonverbal_numerical_cognition_from_reals_to_integers

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* Washington, DC: American Psychological Association.

Gersten, R., Jordan, S., & Flojo, J. (2005). Early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293-304. https://doi.org/10.1177/00222194050380040301

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*, 1202–1242. https://doi.org/10.3102/0034654309334431

Ghazi, S. R., & Ullah, K. (2015). Concrete operational stage of Piaget's cognitive development theory: An implication in learning general science. *Goma University Journal of Research, 31*(1), 78-89. Retrieved from https://pdfs.semanticscholar. org/4a60/604c23944balefe2ad32651c091f62fdf1f.pdf

Ghazi, S. R., & Ullah, K. (2016). Concrete operational stage of Piaget's cognitive development theory: An implication in learning mathematics. *Gamal University Journal of Research, 32*(1), 9-20. Retrieved from https://search-ebscohost.com. ezproxy.barry.edu.login.aspx?direct=true&db=asn&AN=117769147&site=ehost-live&scope=site

Ginsburg, H. P., & Baroody, A. J. (2003). *Test of early mathematics ability* (3rd edition). Austin, TX: Pro.Ed.

Gitter, L. (1967). The promise of Montessori for special education. *Journal of Special Education, 2*(1), 5-13. https://doi.org/10.1177/002246696700200101

Glermain, J. (2008). Montessori math moves from the concrete to the abstract. *Tomorrow's Child.* Retrieved from https://www.xavier.edu/montessori-lab-school/documents/MontessoriMathMovesFromtheConcretetotheAbstract-ParentEdMathNightResource.pdf

Goddard, R., Hoy, R. D., Hoy, W. K., & Woolfolk-Hoy, A. W. (2000). Collective teacher efficacy: Its meaning, measure, and impact on student achievement. *American Educational Research Journal, 37*(2), 479-507. doi:10.3102/00028312037002479

Green, K. B., Gallagher, P. A., & Hart, L. (2018). Integrating mathematics and children's literature for young children with disabilities. *Journal of Early Intervention, 40*(1), 3-19. doi:10.1177/1053815117737339

Gutek, G. L. (2004). *The Montessori Method: The origins of an educational innovation: Including an abridged and annotated edition of Maria Montessori's the Montessori Method*. Lanham, MD: Rowman & Littlefield Publishers.

Hannula-Sormunen, M. M., Lehtinen, E., & Rasanen, P. (2015). Preschool children's spontaneous focusing on numerosity, subitizing, and counting skills as predictors of their mathematical performance seven years later at school. *Mathematical Thinking & Learning, 17*(2/3), 155-177. doi:10.1080/10986065.2015.1916814

Harpe, S.E. (2015). How to analyze Likert and other rating scale data. *Currents in Pharmacy Teaching and Learning, 7*(6), 836-850. https://doi.org/10.1016/j.cptl.2015.08.001

Haq, Z. I., & Alfilfili, H. H. (2015). The efficiency of an educational program based on Montessori curriculum in developing logical thinking in kindergarten children. *European Journal of Research and Reflection in Educational Sciences, 3*(1), 8-20. Retrieved from http://www.idpublications.org/wp-content/uploads/2015/01/THE-EFFICIENCY-OF-AN-EDUCATIONAL-PROGRAM-BASED-ON-MONTESSORI-CURRICULUM-IN-DEVELOPING-LOGICAL-THINKING-IN-KINDERGARTEN-CHILDREN.pdf

Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.

Hewitt, K. (2001). Blocks as a tool for learning: Historical and contemporary perspectives. *Young Children.* Retrieved on April 20, 2017 from http://www.naeyc.org/files /yc/file/Hewitt0101.pdf

Hinton, V. W., Flores, M. M., Schweck, K., & Burton, M. E. (2016). The effects of a supplemental explicit counting intervention for preschool children. *Preventing School Failure, 60*(3), 183-193. doi:10.1080/1049088x.2015.1065-400

Hu, W., Jia, X., Pluckers, J., & Shan, X. (2016). Effects of a critical thinking skills program on the learning motivation of primary school students. *Roeper Review, 38*(2), 70-83. doi:10.1080/02783193.2016.1150374

Hudson, M. E., Zambone, A., & Brickhouse, J. (2016). Teaching early numeracy skills using single switch voice-output devices to students with severe multiple disabilities. *Journal of Developmental Physical Disabilities,* 28, 153-175. doi:10.1007/s10882-015-9451-3

Hunting, R. P. (2013). Part-whole number knowledge in preschool children. *Journal of Mathematical Behavior, 22*, 217-235. doi:10.1016/S0732-3123(03)00021-X

Huntley-Fenner, G. (2001). Children's understanding of number is similar to adults' and rats': Numerical estimation by 5- to 7-year-olds. *Cognition, 78,* B27-B40. doi:0010-02770175

Individuals with Disabilities Education Act, 20 U.S.C. § 1400 (1990). Retrieved from https://files.eric.ed.gov/fulltext/ED343318.pdf

Individuals with Disabilities Education Improvement Act, 20 U.S.C. § 1400 (2004). Retrieved from https://www2.ed.gov/policy/speced/leg/idea/idea.pdf

Itard, J.M. (1802). A historical account of the discovery and education of a savage man: Or, the first developments, physical and moral, of the young savage caught in the woods near Averyon in the year 1798. London: Richard Phillips.

Ivanova, T. (2014). The place of the teacher in the system of pedagogy of Maria Montessori and the contemporary system of education. *Management and Education, 10*(3), 26-32.

Izard, V., Streri, A., & Spelke, E. S. (2014). Toward exact number: Young children use one-to-one correspondence to measure set identity but not numerical equality. *Cognitive Psychology, 72*(2), 27-53. doi:10.1016/j.cogpsycho.2014.01.004

Jacobi-Vessels, J., Brown, T. E., Molfese, V., & Do, A. (2016). Teaching preschoolers to count: Effective strategies for achieving early mathematics milestones. *Early Childhood Education Journal, 44*(1), 1-9. doi:10.1007/s10643-014-0671-4

Jamieson, A., Curry, A., & Martinez, G. (2001). *School enrollment in the United States, social and economic characteristics: October 1999* (Series P20-533). Washington, DC. U.S. Census Bureau. Retrieved from https://cps.oipums.org/cps/resources/cpr/p20-533.pdf

Jelas, Z. M., & Dahan, H. M. (2010). Gender and educational performance: The Malaysian perspective. *Procedia Social and Behavioral Sciences, 7*(C), 720-727. doi:10.1016/j.sbspro.2010.10.098

Jimenez-Fernandez, G. (2015). How can I help my students with learning disabilities in mathematics? *REDIMAT, 5*(1), 56-73. doi:10.4471/redimat.2016.1469

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews,* 15, 60-68. doi:10.1002/ddrr.46

Kamerman, S. B., & Gatenio-Gabel, S. (2007). Early childhood education and care in the united states: An overview of the current policy picture. *International Journal of Child Care and Education policy, 1*(1), 23-24. Retrieved from https://www.childcarecanada.org /sites/default/files/3_Kammerman.pdf

Kayili, G. & Ari, R. (2011). Examination of the effects of the Montessori Method on preschool children's readiness to primary education. *Educational Sciences: Theory and Practice, 11*(4), 2104-2109. Retrieved from https://files.eric.ed.gov/fulltext/EJ962690.pdf

Kayili, G. (2018). The effect of Montessori Method on cognitive temp of Kindergarten children. *Early Child Development and Care, 188*(3), 327-335. doi:10.1080/03004430.2016.1217849

Khaleel, M. (2017). Female students are more likely to get higher grades than male students. *International Journal of Scientific and Research Publications, 7*(3), 378-386. Retrieved from http://www.ijsrp.org/research-paper-0317/ijsrp-p6357.pdf

Khomais, S. F. (2014). Enhancing preschool children's number knowledge: The suitability of an intervention program for Saudi practice. *Early Child Development and Care, 184*(1), 32-49. doi:10.1080/03004430.2013.768238

Kids Count Data Center. (2015). Selected kids count indicators for county in Florida. *Annie E. Casey Foundation*. Retrieved from http://datacenter.kidscount.org/ data/customreports/ 1865/52955296,5342,5347,583,585-586

King, S. A., Lemons, C. J., & Davidson, K. A. (2016). Math interventions for students with autism spectrum disorder: A best-evidence synthesis. *Exceptional Children, 82*(4), 443-462. doi:10.1177/001442915625066

Knudsen, B., Fischer, M. H., & Aschersleben, G. (2015). Development of spatial preferences for counting and picture naming. *Psychological Research, 79*(6), 939-949. doi:10.1007/s00426-014-0623-z

Kose, U., & Arslan, A. (2015). Realizing an optimization approach inspired from Piaget's theory of cognitive development. *Broad Research in Artificial Intelligence & Neuroscience, 6*(1/2), 14-21. Retrieved from https://arxiv.org/pdf/1704.05904.pdf

Kroesbergen, E. H., van t' Noordende, J. E., & Kolkman, M. E. (2014). Training working memory in kindergarten children: Effects on working memory and early numeracy. *Child Neuropsychology, 20*(1), 23-27. doi:10.1080/09297049.2012.736483

Kyttala, M., Kanerva, K., & Kroesbergen, E. (2015). Training counting skills and working memory in preschool. *Scandinavian Journal of Psychology, 56*(40, 363-370. doi:10.1111/sjop.12221

Lagrange, J.B., & Kynigos, C. (2014). Digital technologies to teach and learn mathematics: Context and re-contextualization. *Educational Studies in Mathematics, 85*(3), 381-403. doi:10.1007/s10649-013-9525-z

La Haye, R., & Naested, I. (2014). Mutual interrogation: A celebration of alternative perspectives for visual art and math curriculum. *Canadian Review of Art Education, 41*(2), 185-201. Retrieved from https://www.lkca.nl/informatiebank/mutual-interrogation

Lappan, G., & Schram, P. (1989). Communication and reasoning: Critical dimensions of sensemaking in mathematics. In P. R. Trafton & A. P. Shulte (Eds.). New directions for elementary school mathematics (pp. 14-30). Reston, VA: NCTM.

Laski, E. V., Jor'dan, J. R., Daoust, C., & Murray, A. K. (2015). What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE,* April-June 2015, 1-8. doi:10.1177/2158244015589588

Lee, J., & Md-Yunus, S. (2016). Investigating children's abilities to count and make quantitative comparisons. *Early Childhood Education Journal, 44*(3), 255-262. doi:10.1007/s10643-015-0707-4

Lefa, B. (2014). The Piaget theory of cognitive development: An educational implication. *Educational Psychology,* 1-9. Retrieved from https://www.researchgate.net/

publications/265916960_THE_PIAGET_THEORY_OF_COGNITIVE_DEVELOPMENT_AN_EDUCATIONAL_IMPLICATIONS

Lefmann, T., & Combs-Orme, T. (2013). Early brain development for social work practice: Integrating neuroscience with Piaget's theory of cognitive development. *Journal of Human Behavior in the Social Environment, 23*(5), 640-647. doi:10.1080/10911359.2013.775936

Leongson, J. A., & Limpjap, A. L. (2003). Assessing the mathematical achievement of college freshmen using Piaget's logical operations. Retrieved from http://www.cimt.org.uk/ journal/limjap.pdf

Lewis, K. E. (2016). Difference not deficit: Conceptualizing mathematical learning disabilities. *Journal of Education, 196*(2), 39-62. Retrieved from http://www.jstor.org/stable/ 10.5951/jresematheduc.45.3.0351

Lillard, A. S. (2005). *Montessori: The science behind the genius*. Oxford: Oxford University Press.

Lillard, A. S. (2012). Preschool children's development in classic Montessori, supplemented Montessori, and conventional programs. *Journal of School Psychology, 50,* 379-401. doi:10.1016/j.jsp.2012.01.001

Lillard, A. S., (2013). Playful learning and Montessori education. *American Journal of Play, 5*(2), 157-186. Retrieved from https://files.eric.ed.gov/fulltext/EJ1003949.pdf

Lillard, A., & Else-Quest, N. (2006). The early years: Evaluating Montessori Education. *Montessori Education Science, 313*, 1893-1894. doi:10.1126/science.1132362

Lopata, C., Wallace, N. V., & Finn, K. V. (2005). Comparison of academic achievement between Montessori and traditional education programs. *Journal of Research in Childhood Education, 20*(1), 5-13. doi:1080/025685405095994546

Madu, C. I. (2016). Students' evaluation of mathematics teachers' preparedness for effective instruction (A case study of Kano State secondary school mathematics teachers. *Journal of Research and Method in Education, 6*(3), 10-14. Retrieved from http://www.iosrjournals.org/iosr-jrme/papers/Vol-6%20Issue-3/Version-3/C0603031014.pdf

Manan, S. A., & Khadija-Tul-Kubra, N. (2017). The younger, the better: Idealized versus situated cognitions of educators about age and instruction of English as a second/foreign language in Pakistan. *Language Sciences, 64*(1), 54-68. doi:10.1016/j.langsci.2017.07.002

Manner, J. C. (2006). Montessori vs. traditional education in the public sector: Seeking appropriate comparisons of academic achievement. *Forum on Public Policy.* Retrieved from http://forumonpublicpolicy.com/archivespring07/manner.pdf

Marmasse, N. Bletsas, A., & Marti, S. (2000). Numerical mechanisms and children's concept of numbers. *Massachusetts Institute of Technology Media Laboratory, Version 6.* 1-9. Retrieved from http://alumni.media.mit.edu/~stefanm/society/som_final.html

Martin, T., Smith, S., Brasiel, S., & Sorensen, I. (2017). Online developmental mathematics: Challenging coursework traditions. *Journal of Developmental Education, 40*(3), 1-12. Retrieved from https://files.eric.ed.gov/fulltext/EJ1184233.pdf

Mattoon, C., Bates, A., Shifflet, R., Latham, N., & Ennis, S. (2015). Examining computational skills in prekindergartners: The effects of traditional and digital manipulatives in a prekindergarten classroom. *Early Childhood Research and Practice, 17*(1). Retrieved from http://ecrp.uiuc.edu/v17n1/mattoon.html

Maxim, G. W. (1989). Developing preschool mathematical concepts. *The Arithmetic Teacher, 37*(4), 36-42. Retrieved from https://www.jstor.org/stable/i40053542

Mazzocco, M. M., Myers, G. F., Lewis, K. E., Hanich, L. B., & Murphy, M. M. (2013). Limited knowledge of fraction representations differentiates middle school students with mathematics learning disability (dyscalculia) versus low mathematics achievement. *Journal of Experimental Child Psychology, 115*(2), 371-387. doi:10.1016/j.jeep.2013.01.005

McNeil, N. M., & Uttal, D. H. (2009). Rethinking the use of concrete materials in learning: Perspectives from development and education. *Child Development Perspectives, 3*(3),137-139. https://doi.org/10.1111/j.1750-8606-2009.00093.x

Mendizabal, E., Villagran, M., Guzman, J. I., & Hoyos, A. (2015). Effects of a specific training program for early mathematics learning in early childhood education. *Revista Espanola de Pedagogia, 73*(260), 105-119. Retrieved from https://revistadepedagogia.org/wp-content/uploads/2015/01/efectos-de-la-aplicacion.pdf

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education, 30*(1), 65-88. Retrieved from https://www.jstor.org/stable/749630

Milinkovic, J., & Bogavac, D. (2013). Montessori method as a basis for integrated mathematics learning. *Methodological Horizons, 6*(1). doi:10.32728/mo.06.1.2011.11

Mix, K. S., Smith, L. B., Stockton, J. D., Cheng, Y. L., & Barterian, J. A. (2017). Grounding the symbols for place value: Evidence from training and long-term exposure to base-10 models. *A Journal of Cognition and Development, 18*(1), 129-151. doi:10.1080/15248372.2016.1180296

Mononen, R., Aunio, P., Koponen, T., & Aro, M. (2014). A review of early numeracy interventions for children at risk in mathematics. *International Journal of Early Childhood Special Education, 6*(1), 25-54. Retrieved from https://helda.helsinki.fi/bitstream/handle/10138/232676/20150930191208_intjecse.pdf?sequence=1

Montessori, M. (1936). *The secret of childhood.* B.B. Carter (Ed.). Calcutta: Orient Longmans.

Montessori, M. (1949). *The absorbent mind*. Adyar, India: The Theosophical Publishing House.

Montessori, M. (1964). *Dr. Montessori's own handbook (*Reprint of original publication of 1914*).* Cambridge, MA: Robert Bentley, Inc.

Montessori, M. (1967). *The absorbent mind* (Reprint of original publication). New York: Dell Publishing Co., Inc.

Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives used by K-8 teachers for mathematics instruction: Considering mathematical, cognitive, and pedagogical fidelity. *Contemporary Issues in Technology and*

Teacher Education, 8, 202-218. Retrieved from https://eric.ed.gov/?id=EJ1154970

Mutjaba, T., & Reiss, M. (2016). Girls in the UK have similar reasons to boys for intending to study mathematics post-16 thanks to the support and encouragement they receive. *London Review of Education, 14*(2), 68-72. doi:10.18546/LRE.14.2.05

Nakagawa, S. (2004). A farewell to Bonferroni: The problems of low statistical power and publication bias. *Behavioral Ecology, 15(6),* 1044-1045. doi:10.1093/beheco/arh107

National Association for the Education of Young Children. (2010). Early childhood mathematics: Promoting good beginnings. *NAEYC Position Statement*. Retrieved from https://www.naeyc.org/sites/default/files/globally-shared/downloads/PDFs/resources/ position-statements/psmath.pdf

National Center for Education Statistics, (2018). Children and youth with disabilities. *The Condition of Education.* Retrieved from https://nces.ed.gov/programs/coe/ indicator_cgg.asp

National Council for Teachers of Mathematics. (2003). *Assessment standards for school readiness.* Reston, VA: National Council of Teachers of Mathematics.

National Education Goals Panel. (1999). *The National Education Goals report: Building a nation of learners, 1999.* Washington, DC: U.S. Government Printing Office.

Navarro, J. (2014). Critical aspects of Piaget's conception of numbers. *Pensando Psicologia, 10*(17), 97-101. doi:10.16925/pe.v10i17.788

Nguyen, T., Watts, T. W., Duncas, G. J., Clermente, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool math competencies are most predictive of 5th grade achievement? http://dx.doi.org/10.1016/j.ecreiq.2016.02.003

Office of Head Start. (2017). *History of head start.* Retrieved from https://www.acf.hhs.gov/ ohs/about/history-of-head-start

Office of Special Education and Rehabilitative Services. (2016). 38th annual report to congress on the implementation of the individuals with disabilities education act. *U.S. Department of Education.* Retrieved from https://www2.ed.gov/about/reports/annual/osep/2016/parts-b-c/(-arc-for-idea.pdf

Ojose, B. (2008). Applying Piaget's theory of cognitive development to mathematics instruction. *The Mathematics Educator, 18*(1), 26-30. Retrieved from https://files.eric.ed.gov/ fulltext/EJ841568.pdf

Ongoren, S., & Turcan, A. I. (2009). The effectiveness of Montessori education method in the acquisition of concept of geometrical shapes. *Procedia Social and Behavioral Sciences, 1,* 1163-1166. doi:10.1016/j.sbspro.2009.01.209

Papadakis, S., Kalogiannakis, M., & Zaranis, N. (2017). Improving mathematics teaching in kindergarten with realistic mathematical education. *Early Childhood Education Journal, 45*(3), 369-378. doi:10.1007/s10643-015-0768-4

Passolunghi, M. C., & Costa, H. M. (2016). Working memory and early numeracy training in preschool children. *Child Neuropsychology, 22*(1), 1-18. doi:10.1080/09297049.2014.971726

Peterson, L. A., & McNeil, N. M. (2013). Effects of perceptually rich manipulatives on preschoolers' counting performance: Established knowledge counts. *Child Development, 84*(3), 1020-1033. doi:10.1111/cdev.12028

Perneger, T.V. (1998). What's wrong with Bonferroni adjustments. *British Medical Journal, 316*(7139), 1236-1238. Retrieved from http://www.ncbi.nlm.nih.gov /pmc/articles/PMC1112991/

Piaget, J. (1942). *The child's conception of number.* London: Routledge & Kegan Paul.

Piaget, J. (1965). *The child's conception of number*. New York: Norton.

Pickering, J. (1992). Successful applications of Montessori methods with children at risk for learning disabilities. *Annals of Dyslexia, 42*(1), 90-109. Retrieved from https://www.jstor.org/stable/23767994

Pinhas, M., Donohue, S. E., Woldorff, M. G., & Brannon, E. M. (2014). Electrophysiological evidence for the involvement of the approximate number system in preschoolers' processing of spoken number words. *Journal of Cognitive Neuroscience, 26*(9), 1891-1904. doi:10.1162/jocn_a_00631

Pitcher, N., Goldfinch, J., & Beevers, C. (2002). Aspects of computer-based assessment in mathematics. *Active Learning in Higher Education, 3*(2), 159-176. https://doi.org/10.1177/1469787402003002005

Posid, T., & Cordes, S. (2014). Verbal counting moderates perceptual biases found in children's cardinality judgements. *Journal of Cognition and Development, 16*(4), 621-637. doi:10.1080/15248372.2014.934372

Post, T. (1981). The role of manipulative materials in the learning of mathematical concepts. In selected issues in Mathematics Education (p. 109-131). Berkeley, CA: National Society for the Study of Education and National Council of Teachers of Mathematics, McCutchan Publishing Corporation.

Price, G. R. (2013). Dyscalculia: Characteristics, causes, and treatments. *Numeracy, 6*(1), 1-16. http://dx.doi.org/10.5038/1936-4660.6.1

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal, 50*(1), 178-209. doi:10.3102/0002831212465332

Purpura, D. J., & Lonigan, C. J. (2015). Early numeracy assessment: The development of the preschool numeracy scales. *Early Education Development, 26*(2), 286-313. doi:10.1080/10409289.2015.991084

Purpura, D. J., & Napoli, A. (2015). Early numeracy and literacy: Untangling the relation between specific components. *Mathematical Thinking and Learning, 17*(2-3), 197-218. doi:10.1080/10986065.2015.1016817

Purpura, D. J., Reid, E. E., Eiland, M. D., & Baroody, A. J. (2015). Using a brief preschool early numeracy skills screener to identify young children with mathematics difficulties. *School Psychology Review, 44*(1), 41-59. doi:10.17105/SPR44-1.41-59

Reardon, S. F., & Portilla, X. A. (2016). Recent trends in income, racial, and ethnic school readiness gaps at kindergarten entry. *AERA Open*, doi:10.1177/2332858416657343

Reed, M. (2000). A comparison of the place value understanding of Montessori and non-Montessori elementary school students. (Doctoral dissertation). Retrieved from https://etd.ohiolink.edu/pg_10?0::NO:10:P10_ACCESSION_NUM:osu1391509130

Reedal, K. E. (2010). Jean Piaget's cognitive development theory of mathematics education. *Summation,* 16-20. Retrieved from http://ripon.edu/macs/summation

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology, 3*(1), 41-95. Hillsdale, NJ: Erlbaum

Reid, K. (2016). Counting on it: Early numeracy development and the preschool child. *Australian Council for Educational Research,* 2, 1-11. Retrieved from http://research. acer.edu.au/cgi/viewcontent.cgi?article=1020&context=learning_processes

Reid, K., & Andrews, N. (2016). Longitudinal literacy and numeracy study: Transitions from preschool to school: Fostering understanding of early numeracy development. *Australian Council for Educational Research*, 1-12. *Journal of Mathematics Teachers Education, 5*, 205-233. Retrieved from http://research.acer.edu.au/cgi/viewcontent.cgi?article= 1028&context=monitoring_learningcommunity

Rimm-Kaufman, S. E., & Sawyer, B. E. (2004). Primary-grade teachers self-efficacy beliefs, attitudes towards teaching, and teaching practice priorities in relation to the responsive classroom approach. *The Elementary School Journal, 104*(4), 321-341. Retrieved from https://pdfs.semanticscholar.org/8701/35ffeefca9flfd90e18413f61a93eb633be7pdf?_ga=2.243154305.229899262.1574607301-1120161939.1574607301

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition & Development, 14*(3), 376.396. doi:10.1080/15248372.2012.689897

Rosli, R., Goldsby, D., & Capraro, M. M. (2015). Using manipulatives in solving and posing mathematical problems. *Creative Education,* 6, 1718-1725. doi:10.4236/ce.2015.616173

Roth, W. M. (2017). Astonishment: A post-constructivist investigation into mathematics as passion. *Educational Studies in Mathematics, 95*(2017), 97-111. doi:10.1007/s10649-016-9733-4

Schneider, M. Beeres, K., Coban, L., Merz, S., Schmidt, S. S., Stricker, J., & Smedt, B. D. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3), 1-16. doi:10.1111/desc.12372

Sequin, E. (1846). Treatment moral, hygiene et education des idiots et des autres enfants arrieres. Oxford University: Bailliere.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). Experimental and quasi-experimental designs for generalized causal inference. Retrieved from https://moodle2.units.it/ pluginfile.php/132646/mod_resource/content/1/ Estratto_ShadishCookCampbellExperimental2002.pdf

Shephard, L. A., & Smith, M. L. (1988). Escalating academic demand in kindergarten: Counterproductive policies. *Elementary School Journal, 89*(20), 135-145. Retrieved from https://www.colorado.edu/education/sites/default/files/attached-files/attached-files/Escalating%20Academic%20Demand.pdf

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics. *Journal of Mathematics Teacher Education, 5*(3), 205-233. doi:10.1023/A:1020134209073

Shusterman, A., Slusser, E., & Odic, D. (2016). Acquisition of the cardinal principle coincides with improvement in approximate number system acuity in preschoolers. *PLoS ONE, 11*(4), 1-22. doi:10.1371/journal.pone.0153072

Sophian, C. (2009). Numerical knowledge in early childhood. *Encyclopedia of Early Childhood Development,* 1-7. Retrieved from http://www.child-encyclopedia.com/documents/ SophianANGxp.pdf

Soydan, S. (2015). Analyzing efficiency of two different methods involving acquisition of operational skills by preschool children. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(1), 129-138. doi:10.12973/Eurasia.2014.1036a

Steedly, K. Dragoo, K., Arafeh, S., & Luke, S. D. (2008). Effective mathematic instruction. *National Dissemination Center for Children with Disabilities, 3*(1), 1-11. Retrieved from http://www.parentcenterhub.org/wp-content/uploads/ repo_items/eemath.pdf

Steffe, L. P. (2016). Toward a model of constructivist mathematics teaching. *Radical Constructivism, 12*(1), 75-77. Retrieved from http://constructivist.info/12/ 1/059.borg

Stipek, D., Macgyvers, V., & Givvin, K.B. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2001), 213-226. doi:10.1016/S0742-051X(00)00052-4

Szu ‘cs, D., & Goswani, U. (2013). Developmental dyscalculia: Fresh perspectives. *Trends in Neuroscience and Education, 2,* 33–37. http://dx.doi.org/10.1016/j.tine.2013.06.004

Talbot, A. B., Ahmad, A. C., & Ghazali, M. (2013). Examine counting procedure among students with mild intellectual disability: A case of Penang Malaysia. *International Education Studies, 6*(5), 33-46. doi:10.5539/ies.v6n5p33

Takunyaci, M. & Takunyaci, M. (2014). Preschool teacher’s mathematics teaching efficacy belief. *Procedia Social and Behavioral Science, 152*(7), 673-678. https://doi.org.10.1016/j.sbspro.2014.09

Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Center for Research in Mathematics and Science Education, 1-11.* Retrieved from http://pat-thompson.net/PDFversions/1994Concrete.pdf

Tichenor, M., Welsh, A., Corcoran, C., Piechura, K. & Heins, E. (2016). Elementary girls’ attitudes toward mathematics in mixed-gender and single-gender classrooms *Education, 137*(1), 93-100. Retrieved from https://eric.ed.gov/?id=EJ1112181

Toll, S. W., & Van Luit, J. E. (2014). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children*. https://doi.org/10.1177/0014402914532233

Tunyiova, M., & Sarmany-Schuller, I. (2016). Somatic markers in decision-making with regard to the cognitive development of 6- to 14- year-old children. *Studia Psychologica, 58*(1), 32-46. doi:10.21909/sp.2016.01.705

Tzur, R., Johnson, H. L., McClintock, E., Kenney, R. H., Xin, Y. P., Si, L., & Jin, X. (2013). Distinguishing schemes and tasks in children’s development of multiplicative reasoning. *PNA*, *7*(3), 85–101. Retrieved from https://www.math.purdue.edu/~rhkenney/Kenney_ Homepage/Links_to_Publications_files/PNA_Published.pdf

U.S. Census Bureau. (2015). Table A - 1. School Enrollment of the Population 3 Years Old and Over, by Level and Control of School, Race, and Hispanic Origin: October 1955 to 2014. Retrieved from http://www.census.gov/ hhes/school/data/cps/historical/

United States Census. (2018). Quick facts: Pembroke Pines. Retrieved from https://www.census.gov/quickfacts/fact/table/pembrokepinescityflorida/PST045218

U.S. Department of Education. (2009). *The early learning challenge fund.* Retrieved from https://www2.ed.gov/about/inits/ed/earlylearning/elcf-factsheet.html

U.S. Department of Education. (2018). *Building the legacy: IDEA 2004*. Retrieved from http://idea.ed.gov/

Uttal, D. H., Scudder, K. V., & Deloache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*(1997), 37-54. Retrieved from http://faculty.virginia.edu /deloache/Manipulatives%20as%20symbols %20(1997).pdf

Van Herwegen, J., Costa, H.M., Nicholson, B., & Donlan, C. (2018). Improving number abilities in low achieving preschoolers: Symbolic versus non-symbolic training programs. *Research in Developmental Disabilities, 77*(3), 1-11. doi:10.1016/j.ridd.2018.03.011

Varol, F., & Farran, D. C. (2006). Early mathematical growth: How to support young children's mathematical development. *Early Childhood Education Journal, 33*(6), 381-387. doi:10.1007/s10643-006-0060-8

Voustina, C. (2016). Oral counting sequences: A theoretical discussion and analysis through the lens of representational redescription. *Educational Studies in Mathematics, 93*(2), 175-193. doi:10.1007/s10649-016-9705-8

Wang, M. C., Haertel, G. D., & Walberg, H. J. (1993). Toward a knowledge base for school learning. *Review of Educational Research, 63*, 249-294. https://doi.org/10.3102/00346543063003249

Waxman, H. C., & Huang, S. L. (1997). Classroom instruction and learning environment differences between effective and ineffective urban elementary schools for African American students. *Urban Education, 32*(1), 7-44. https://doi.org/10.1177/0042085997032001002

West, J. (2017). National longitudinal studies of kindergarten children.: Historical context and ongoing contributions. *AERA Open, 3*(2), 1-18. doi:10.1177/2332858417701684

West Virginia Department of Education. (2016). Building numeracy through mathematical progressions: Birth to grade 3. Retrieved from https://wvde.us/wp-content/uploads/2017 /10/learning-progression-birth-to-grade-3.pdf

Williams-Pierce, C. C. (2011). Five key ingredients for improving student motivation. *Research in Higher Education Journal, 11*. Retrieved from http://aabri.com/manuscripts/11834.pdf

Wolfe, J. (2002). *Learning from the past: Historical voices in early childhood education* (2nd ed.). Alberta, CA: Mayerhoper, Piney Branch Press.

Wolfgang, C. H., Stannard, L. L., & Jones, T. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. *Journal of Research in Childhood Education, 15*(2), 173-180. Retrieved from https://www.niu.edu/p20network/+pdfs/work-groups/b-3/Wolfgang_Stannard_ Jones_Blck-play-perf-predictor.pdf

Woodward, J., & Tzur, R. (2017). Final commentary to the cross-disciplinary thematic special series: Special education and mathematics education. *Learning Disability Quarterly, 40*(3), 146-151. doi:10.1177/0731948717690117

Wong, W. I. (2017). The space-math link in preschool boys and girls: Importance of mental transformation, targeting accuracy, and spatial anxiety. *British Journal of Developmental Psychology, 35*(2), 249-266. doi:10.1111/bjdp.12161

Xin, Y. P., Liu, J., Jones, S., Tzur, R., & Sin, L. (2016). A preliminary discourse analysis of constructivist-oriented mathematics instruction for a student with learning disabilities. *Journal of Educational Research, 109*(4), 436-447. doi:10.1080/00220671.2014.979910

Young-Loveridge, J. (2001). Helping children move beyond counting to part-whole strategies. *Teachers and Curriculum, 5*, 72-78. http://doi.org/10.1177/183693910202700408

Zadnik, K., & Koren, D. (2017). Montessori pedagogy and its role in music theory in music school. *Glasbeno, 27*(1), 175-190. Retrieved from http://search.ebscohost.com/ login.aspx?direct=true&site=eds-live&db=rih&AN=A1282118

Zhu, X., Chen, Y., Li, Y., & Deng, Z. (2017). Automatic non-symbolic numerosity processing preschoolers. *PLoS ONE, 12*(6), 1-17. https://doi.org/10.1371/journal.pone.0178396

Zur, O., & Gelman, R. (2004). Young children can add and subtract by predicting and checking. *Early Childhood Research Quarterly, 19*, 121-137. doi:10.1016/j.ecresq.2004.01.003

Appendices

Appendix A

Student Participant Demographic Form

School # _______ Class Name: _____________					
		Gender	**Ethnicity**	**Age**	**Disability Category (DD/ND)**
Student Name	**#**	**(M/F)**	**(W, B, H, A, O)**	**3, 4, 5**	

Appendix B

Test of Early Mathematics Permission Form

pro·ed
An International Publisher

page 1

Approval of Permission to Use PRO-ED Test Material

April 11, 2019 Reference Permission Request #[redacted]

Ms. Katrina Azevedo-Pinillos
Barry University
[redacted]

For permission to use the Form A and Form B of the Test of Early Mathematics - Third Edition (TEMA-3) by Ginsburg, Baroody, , Austin: PRO-ED. Kit 10880. Number of copies: 640 No fee assessed.

USAGE: Research for Master's Thesis or Dissertation

This dissertation study will investigate the impact of the Montessori Short Bead Stairs (SBS) on preschool children with/without developmental delays knowledge and performance of counting, identification, cardinality, and quantity of numbers 1-10. In a randomized alternative-treatment with pretest design, 16 classroom teachers and their classes from 3 preschools in Broward will be randomly assigned to one of two treatment groups: traditional method (A) and Montessori SBS (B). In a six-week study, teachers will be surveyed on classroom techniques and methods, and 320 preschool children will be assessed for pre- and post-test results using a portion of the Test of Early Mathematics Ability, 3rd Edition. Children in Treatment A will receive daily traditional methods of learning and children in Treatment B will receive daily Montessori SBS lessons. Data analysis will be conducted to compare teacher techniques and methods, and pre- and post-assessment results for children with/without developmental delays using interventions.

LIMITATIONS:

Permission is granted to utilize Form A and B of the TEMA-3 in this dissertation study. A complimentary test kit will be sent to the requester and she is responsible for purchasing any additional forms needed. Requester agrees to not copy, modify, or otherwise alter any components of the test.

PAYMENT: No fee assessed.

Total Paid: $

APPROVAL:

The foregoing application is hereby approved provided that the form of credit and copyright notice, as specified in the sixth edition of the *Publication Manual of the American Psychological Association* or an equally recognized format, gives full identification of author, publisher, copyright date, and title and states, "Used with Permission." This permission is solely for adaptation to non-original formats and should not be construed as a transfer of any rights,

Approval of Permission to Use PRO-ED Test Material

April 11, 2019 Reference Permission Request [redacted]

title or interest in the PRO-ED publication. This permission includes the right to approve, without charge, the publication or transcription in Braille, large print, audio or other formats, only for the use by print impaired individuals or to accommodate student IEP requirements and only if such an edition is not for commercial use. Should PRO-ED, Inc. in its sole discretion, determine the use of our material by you, the client, is contrary to the original intent as we understood it in your letter requesting permission, we reserve the right to demand that you cease and desist in your use of PRO-ED, Inc.'s material and remove it from the marketplace. PRO-ED makes no representations and warranties about the validity or reliability of the Licensed Material or its appropriateness or effectiveness with respect to your specific use. You agree to defend and indemnify PRO-ED, Inc. from any claims made against PRO-ED, Inc. on account of your use of the Licensed Material. By accepting this agreement, you confirm that the Licensed Material will not be used in pharmaceutical research of any kind.

This permission is for one time use only, is not transferable, and terminates Expires at the conclusion of this study. or when the above material goes out of print; whichever comes first.

Approved by PRO-ED, Inc. Representative

[redacted]

Tests Permissions Department
PRO-ED, Inc.
April 11, 2019

PRO-ED, Inc. Tax ID: [redacted]

Appendix C

Teacher Survey Questions

Directions: Using the Likert Rating Scale, read and answer each question based on your teaching styles and methods.

1. I taught lessons daily for 10 minutes during the 3-week study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

2. All students understood the lessons taught.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

3. I am confident that I taught the lessons effectively.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

4. All students learned how to count orally 1-10.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

5. All students learned to understand one-to-one correspondence.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

6. I do not plan to use the strategies and lessons in my classroom after the study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

7. I taught the lessons for more than 10 minutes daily during the 3-week study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

8. Most students had difficulty understanding the lessons taught.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

9. I struggled with teaching the lessons.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

10. Most students had difficulty learning to count orally 1-10.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

11. Most students struggled to understand one-to-one correspondence.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

12. These strategies learned will be helpful in my classroom after the study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

13. I taught the lessons for less than 10 minutes daily during the 3-week study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

14. I needed more practice in teaching the lessons before implementing effectively.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

15. I was confident in teaching the lessons.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

16. I plan to use the strategies and lessons taught in my classroom after the study.

1 = *Strongly Disagree* **2** = *Disagree* **3** = *Neutral* **4** = *Agree* **5** = *Strongly Agree*

Appendix D

Invitational Email Letter to Teachers

Dear Preschool Teachers,

You and your students are invited to participate in a dissertation research project. The title of the study is Montessori Short Bead Stairs Effects, Numeracy Skills, and Preschool Children With and Without Developmental Delays. I am conducting this research in my role as a PhD student in Barry University's Adrian Dominican School of Education. The study findings will be useful in the field of education. The aim of the research is to investigate the impact of using Montessori Short Bead Stairs (used to teach students counting numbers 1-10) or traditional mathematical lessons on preschool children with and without developmental delays in numeracy skills, as measured by their ability to count, to identify numbers (one-to-one correspondence), order numbers (cardinality), and identify the quantity of numbers.

As a preschool teacher, you and your class are invited to participate in this study. All teachers who agree to participate are invited to sign and return the attached consent form via email to myself and attend a one-day training session provided by me on one of the two dates and times listed below. For the study, all teachers will be assigned to one of two conditions, either traditional math lessons or Montessori Short Bead Stairs lessons. You will then be asked to participate in a training dedicated to the type of lessons you will be teaching for the four weeks of the study (either traditional or Montessori Short Bead Stairs). During the training you will learn the process to implement the lessons to which you have been assigned and practice for mastery before implementing in the classroom. All teachers who choose to participate will serve as a gatekeeper to enroll their own students in the study, will be asked to implement the randomly assigned curriculum lessons, and will be asked to complete a Teacher Survey using Survey Monkey that will be emailed to you. I will provide you with all recruitment materials for students and procedures during the training. The training will consist of: an explanation of the study, the process that you will follow in teaching students during the five weeks of the study, the implementation of the intervention, and data collection.

Each training will take approximately 3 hours for one of two days. Each lesson implemented will take approximately 10 minutes daily (includes instruction and student practice) for 3 weeks. The survey will take approximately 5-8 minutes to complete. Once consent forms are received from classroom teachers, teachers will send home with the child a recruitment flyer for parents and a parent consent form. Parents who agree to allow their child to participate in this study will return the signed consent form to you as the classroom teacher. You will be provided with an envelope in which all collected consent forms will be placed inside and retuned to me.

You as the teacher will serve as a gatekeeper (one who collects demographic information and consent forms and returns to me), and as a participant (one who implement a treatment and will complete a survey at the completion of the study).

Your consent to participate in this study is strictly voluntary and should you and/or your students decline to participate, there will be no adverse effects on you or your student child. If you are interested in participating, please respond to this email by _____(Date)_____.

If you have any questions or concerns regarding the study **or** your students' participation in the study, you may contact me, Katrina Azevedo-Pinillos ***at*** katrina.azevedo-pinillos@mymail.barry.edu or 786-303-2443, or my advisor, Dr. Judy Harris-Looby at jharrislooby@barry.edu or 305-899-3709 or the Institutional Review Board point of contact, Jasmine Trana at jtrana@barry.edu or 305-899-3020.

Thank you in advance for your participation,
Ms. Katrina Azevedo-Pinillos, *M.S. in Education*
Barry University

Appendix E

Barry University Teacher Consent Form

Your participation in a dissertation research project is requested. The title of the study is Montessori Short Bead Stairs, Numeracy Skills, and Preschool Children With and Without Developmental Delays. I am conducting this research in my role as a PhD student in Barry University's Adrian Dominican School of Education. The study findings will be useful in the field of education. The aim of the research is to investigate the impact of using Montessori Short Bead Stairs (used for counting numbers 1-10) and traditional methods of learning numeracy skills as measured by preschoolers' ability to count, to identify numbers (one-to-one correspondence), order numbers (cardinality), and identify the quantity of numbers. The study will include preschool children with and without developmental delays.

Your participation will consist of: attending a training, implementing either Montessori Short Bead Stairs lessons or traditional lessons daily for 3 weeks, responding to an online survey provided via email, and providing demographic data about your students.

Each training will take approximately 3 hours for one of two days. Each lesson implemented will take approximately 10 minutes daily (includes instruction and student practice) for 3 weeks. The survey will take approximately 5-8 minutes to complete. Once consent forms are received from classroom teachers, teachers will send home with the child a recruitment flyer for parents and a parent consent form. Parents who agree to allow their child to participate in this study will return the signed consent form to you as the classroom teacher. You will be provided with an envelope in which all collected consent forms will be placed inside and retuned to me.

It is estimated that a total of 320 children and 16 teachers will participate in this study.

The consent to be a research participant is strictly voluntary and should you decline to participate, or should you choose to drop out at any time during the study, there will be no adverse effects on you or your students.

There are no known risks to participate in this study. Although there are no direct benefits to you or your students, your participation in this study may help our understanding of early numeracy skill development.

As a research participant, the information that you provide will be held in confidence to the extent permitted by law. Any published results of the research will refer **to** group averages only and no names will be used in the study. Data will be protected in accordance with the regulations of the Barry University Institutional Review Board, which oversees all University research.

If you have any questions or concerns regarding the study **or** your student's participation in the study, you may contact me, Katrina Azevedo-Pinillos ***at*** Katrina.azevedo-pinillos@mymail.barry.edu or 786-303-2443, or my advisor, Dr. Judy Harris-Looby at jharrislooby@barry.edu or 305-899-3709, or the Institutional Review Board point of contact, Ms. Jasmine Trana at jtrana@barry.edu or 305-899-3020.

If you are satisfied with the information provided and are willing to participate in this research, please indicate your consent by signing this consent form.

Voluntary Consent
I acknowledge that I have been informed of the nature and purposes of this study by the research investigator, that I have read and understand the information presented above, and that I have received a copy of this form for my record.

I give my voluntary consent to participate in this study.

Signature of Teacher Date Signature of Researcher Date

Appendix F

Barry University Recruitment Flyer for Parents

Barry University

A Research Study for Preschool Students

This study will investigate the effects of Montessori Short Bead Stairs as compared to traditional methods of learning counting numbers 1-10 in 3 – 5-year-old children with and without developmental delays.
This study will be under the direction of PhD student Katrina Azevedo-Pinillos from Barry University's Adrian Dominican School of Education.

**Your child's classroom has been selected to participate in this study.
If you are interested in having your child participate,
please complete the consent form provided
by the classroom teacher and return as soon as possible.**

Teachers who have signed a consent form and agreed to participate in this study have been provided with parent consent forms for each child in their class. Participating teachers will be sending home a parent consent form. If you are interested in your child participating in this study, please sign and return the attached consent form to your classroom teacher as soon as possible.

Teachers will serve as gatekeepers (teachers will collect demographic information and consent forms and provide it to the researcher) and participants (teachers will be trained in the math instruction, will implement the instruction daily for 3 weeks, and will complete a survey about how they implemented the instruction).

Your consent to have your child participate in this study is strictly voluntary. Should you and/or your child decline to participate, there will be no adverse effects on you or your child.

The study involves:

- 1 pretest and 1 posttest of early math skills (approximately 10 minutes each)

- 3 weeks of classroom instruction using traditional methods OR Montessori Short Bead Stairs (a hands-on method in Montessori that uses concrete materials to teach the skills)

For more information, please contact:

PhD Student: Katrina Azevedo-Pinillos *at* katrina.azevedo-pinillos@mymail.barry.edu
Advisor: Dr. Judy Harris-Looby at jharrislooby@barry.edu or (305)-899-3709
Institutional Review Board point of contact: Jasmine Trana at jtrana@barry.edu or (305) 899-3020.

Appendix G

Barry University Parental Informed Consent Form

Your child's participation in a dissertation research project is requested. The title of the study is Montessori Short Bead Stairs, Numeracy Skills, and Preschool Children With and Without Developmental Delays. I am conducting this research in my role as a PhD student in Barry University's Adrian Dominican School of Education. The study findings will be useful in the field of education. The aim of the research is to investigate the impact of using the Montessori Short Bead Stairs (used for counting numbers 1-10), compared to traditional methods of learning numeracy skills, as measured by preschoolers' ability to count, to identify numbers (one-to-one correspondence), order numbers (cardinality), and identify the quantity of numbers. The study will include preschool children with and without developmental delays.

Classrooms that participate in the study will be randomly assigned to one of two study groups. One group will continue to learn math concepts through traditional methods. The second group will learn math concepts using the Montessori Short Bead Stairs. During the first and last weeks of the study, each of the students will take a test of math ability. For this test, I will meet individually with your child in your child's classroom for approximately 10 minutes to ask oral questions that assess your child's math skills. The study will last for approximately 3 weeks with daily math lessons provided by your child's regular teacher. All math lessons will take approximately 10 minutes each day.

We anticipate the number of participants to be 320 students and 16 teachers.

The consent to be a research participant is strictly voluntary and should you decline to allow your child to participate or should your child choose to drop out at any time during the study, there will be no adverse effects on you or your child.

There are no known risks to participate in this study. Although there are no direct benefits to your child, his/her participation in this study may help our understanding of how children develop math concepts and counting skills.

As a research participant, the results of your child's pre and posttest will be held in confidence to the extent permitted by law. Any published results of the research will refer to group averages only and no names will be used in the study. Data will be protected in accordance with the regulations of the Barry University Institutional Review Board, which oversees all University research.

If you have any questions or concerns regarding the study or your student's participation in the study, you may contact me, Katrina Azevedo-Pinillos *at* Katrina.azevedo-pinillos@mymail.barry.edu or 786-303-2443, or my advisor, Dr. Judy Harris-Looby at jharrislooby@barry.edu or 305-899-3709 or the Institutional Review Board point of contact, Ms. Jasmine Trana at jtrana@barry.edu or (305)899-3020.

If you are satisfied with the information provided and are willing to allow your child to participate in this research, please indicate your consent by signing this consent form.

Voluntary Consent
I acknowledge that I have been informed of the nature and purposes of this study by the research investigator, that I have read and understand the information presented above, and that I have received a copy of this form for my record.

I give my voluntary consent to allow my child to participate in this experiment.

_______________________ ______________ ____________________________

Signature of Parent   Date   Signature of Researcher   Date

Appendix H

Traditional Training Materials

Implementation Plan and Process
Traditional Group

Week 1
Pre-Test

Week 2				
DAY 1	**DAY 2**	**DAY 3**	**DAY 4**	**DAY 5**
Worksheet 1	Worksheet 2	Worksheet 3	Worksheet 4	Worksheet 5

*SBS = Montessori Short Bead Stairs

Week 3				
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
Worksheet 6	Worksheet 7	Worksheet 8	Worksheet 9	Worksheet 10

Week 4				
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
Match 1-10 Worksheet		Color 1- 10 Worksheet		Write #'s on worksheet 1-10

Week 6
Post-Test

Traditional Materials

- Worksheet 1- 10
  - Show, write and color
- Matching 1-10 Worksheet
- Color 1-10 Worksheet
- Write #'s 1-10 Worksheet

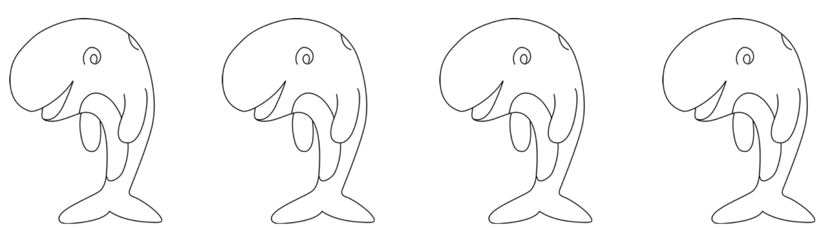
Worksheet 1: Number One

Draw 1 circle:

Trace and write the number:

1	1	1	1	1

Color 1 picture:



Appendix I

Montessori Training Materials

Implementation Plan and Process
Montessori Group

Week 1
Pre-Test

Week 2				
DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Introduce SBS with Number Cards (without Worksheet)	Child practices SBS with number cards (without worksheet)	Child colors over Counting Worksheet 1.	Child colors Counting Worksheet 2	Child reviews SBS with number cards.

*SBS = Montessori Short Bead Stairs

Week 3				
DAY 6	DAY 7	DAY 8	DAY 9	DAY 10
Child uses SBS over SBS 1 Worksheet.	Child colors SBS Worksheet 2	Child matches SBS and matches with card materials	Child completes SBS Worksheet 3 with materials.	Child completes SBS Worksheet 3 without materials.

Week 4				
DAY 11	DAY 12	DAY 13	DAY 14	DAY 15
Child uses number cards and orders the numbers with SBS.	Child order number cards with SBS.	Child completes the Tracing 1 Worksheet.	Child completes Tracing 2 Worksheet WITH materials.	Child completes Tracing 2 worksheet WITHOUT materials.

Week 6
Post-Test

DISSERTATION STUDY
Teacher Training

Title of Dissertation Study	Montessori Short Bead Stairs, Developmental Delays, and Numeracy Skills on Preschoolers
Participants	Children ages 3-5 with and without disabilities Preschool Teachers
Assessment	Test of Early Mathematics-3rd Ed. (TEMA-3) Subsets

After you have completed the 3-week study, the Teacher Survey will be emailed to you for completion.

Montessori Lesson Directions:

MATERIALS:

- a working tray
- mat or small felt
- Montessori colored short bead stairs
- Small wooden number cards
- Short Bead Stairs printables

PROCEDURES

INTRODUCING NUMBERS 1-9

1. Take the beads out of the container and randomly place them on the mat.
2. Start counting by first taking the red (1) bead and count “this is one”.
3. Place it back on the mat, and take the green (2). Say “this is two” as you count the beads and place it back on the mat but this time under the red bead.
4. Continue to work with the rest of the beads until you form a pyramid.
5. Follow up with a 3-Period Lesson:
    a. This is _________.
    b. Where is __________.
    c. Point to __________.
    d. Give me ________.
    e. What is this? _________

DO THIS ONE TIME WITH EVERY CHILD. REPEAT FOR CHILDREN WHO MAY NEED ADDITIONAL SUPPORT.

ASSOCIATING THE QUANTITY TO NUMBER SYMBOLS

Now, the second part of the work is associating the quantity to the number symbol. This time, we’re doing it in a linear form.

1. From the pyramid or stairs, take the red bead from the top, place it on the left side (work from left to right) and say “this is one”.
2. Then take the numerical 1 (Montessori small wooden number cards) and place it beside the red bead.
3. Point to the number symbol and say “this says 1”.
4. Continue until you reach 10.
5. Follow up with a 3-Period Lesson

DO THIS ONE TIME WITH EVERY CHILD. REPEAT FOR CHILDREN WHO MAY NEED ADDITIONAL SUPPORT.

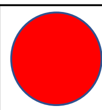
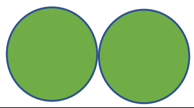
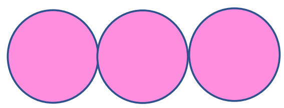
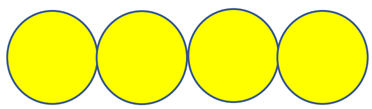
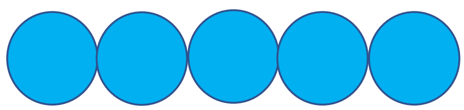
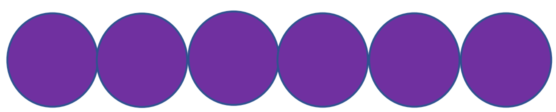
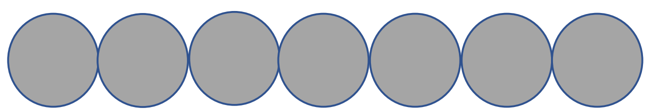
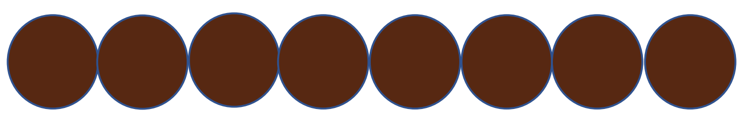
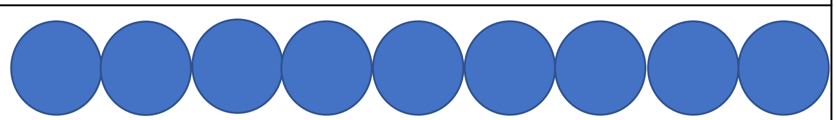
DISSERTATION MATERIALS

MONTESSORI SHORT BEAD STAIRS, DEVELOPMENTAL DELAYS, NUMERACY SKILLS ON PRESCHOOLERS

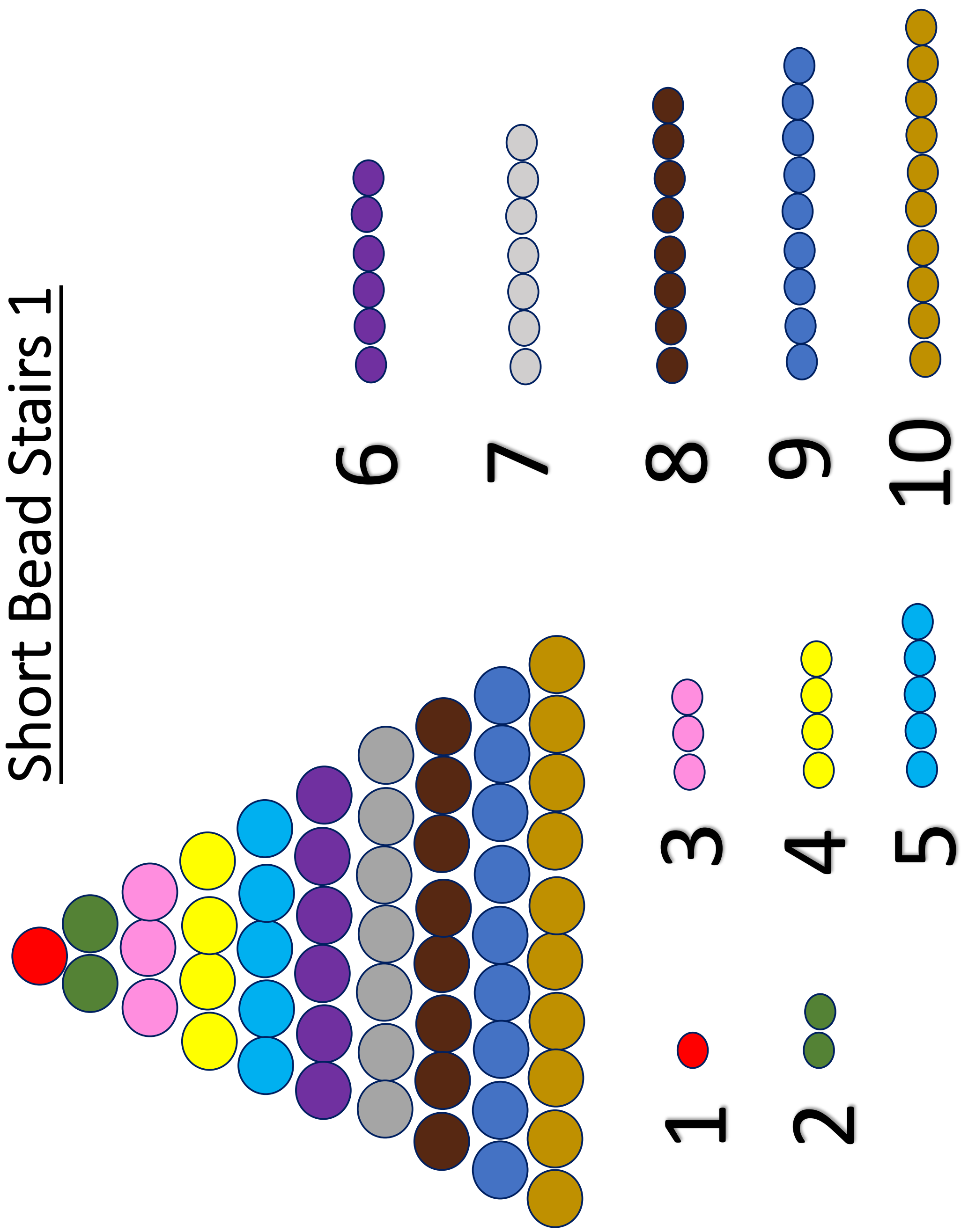
- Short Bead Stairs Counting 1 Worksheet with Numerical Combinations
- Short Bead Stairs Counting 2 Worksheet with Numerical Combinations
- Short Bead Stairs 1 Worksheet
- Short Bead Stairs 2 Worksheet
- Short Bead Stairs 3 Worksheet
- Short Bead Stairs Tracing 1
- Short Bead Stairs Tracing 2

<u>SHORT BEAD STAIRS AND COUNTING 1</u>

With Numerical Combinations

1	
2	
3	
4	
5	
6	
7	
8	
9	

DISSERTATION STUDY - MONTESSORI SHORT
BEAD STAIRS K.AZEVEDO-PINILLOS 2019



DISSERTATION STUDY - MONTESSORI SHORT BEAD STAIRS K.AZEVEDO-PINILLOS 2019

Appendix J

Sample Participant Data Collection Form

Participant #	Pre- Test %				Post-Test %			
	C	IN	O	QN	C	IN	O	QN
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								

Dependent Variables:
C (Counting)
IN (Identifying Numbers)
O (Ordering Numbers)
QN (Identifying Quantity of Numbers)

Barry University
Division of Academic Affairs

Institutional Review Board
11300 NE 2nd Avenue
Miami, FL 33161
P: 305.899.3020 or 1.800.756.6000, ext. 3020
F: 305.899.3026
www.barry.edu

Research with Human Subjects
Protocol Review

Letter of Approval as Exempt

Date:	February 21, 2019
Protocol Number:	1385963-1
Title:	Montessori Short Bead Stairs, Numeracy Skills, and Preschool Children With or Without Developmental Delays
Name:	Katrina Azevedo-Pinillos
Faculty Sponsor:	Dr. Judy Harris- Looby

Dear Ms. Azevedo-Pinillos:

On behalf of the Barry University Institutional Review Board (IRB), you have been granted final approval for this study as exempt from further review.

As principal investigator of this protocol, it is your responsibility to make sure that this study is conducted as approved by the IRB. Any modifications to the protocol or consent form, initiated by you or by the sponsor, will require prior approval, which you may request by completing a protocol modification form.

It is a condition of this approval that you report promptly to the IRB any serious, unanticipated adverse events experienced by participants in the course of this research, whether or not they are directly related to the study protocol.

The **approval** granted expires on February 21, 2020. Should you wish to maintain this protocol in an active status beyond that date, you will need to provide the IRB with and IRB Application for Continuing Review (Progress Report) summarizing study results to date.

If you have questions about these procedures, or need any additional assistance from the IRB, please call the IRB point of contact, Mrs. Jasmine Trana at (305)899-3020 or send an e-mail to **fperez@barry.edu**.

Sincerely,

Fernando Perez, PhD
Chair, Institutional Review Board
Barry University
Department of Sociology & Criminology
11300 NE 2nd Avenue
Miami Shores, FL 33161

****

Note: The investigator will be solely responsible and strictly accountable for any deviation from or failure to follow the research protocol as approved and will hold Barry University harmless from all claims against it arising from said deviation or failure.

Barry University
Division of Academic Affairs

Institutional Review Board
11300 NE 2nd Avenue, Miami, FL 33161
P: 305.899.3020 or 1.800.756.6000, ext. 3020
F: 305.899.3026
www.barry.edu

Research with Human Subjects
Protocol Review

LETTER OF APPROVAL OF MODIFICATION

Date:	Oct 01, 2019
Protocol Number:	1385963-2
Title:	Montessori Short Bead Stairs, Numeracy Skills, and Preschool Children With and Without Developmental Delays
Name:	Katrina Azevedo-Pinillos
Faculty Sponsor:	Dr. Gerene Starratt/Dr. Judy Harris-Looby
Protocol original approval:	Feb 22, 2019
Expiration Date:	Feb 22, 2020

Dear Ms. Katrina Azevedo-Pinillos:

On behalf of the Barry University Institutional Review Board (IRB), I have granted approval of the Modification Request noted above.

As principal investigator of this protocol, it is your responsibility to make sure that this study is conducted as approved by the IRB. Any modifications to the protocol or consent form, initiated by you or by the sponsor, will require prior approval, which you may request by completing a protocol modification form.

It is a condition of this approval that you report promptly to the IRB any serious, unanticipated adverse events experienced by participants in the course of this research, whether or not they are directly related to the study protocol.

Should you wish to maintain this protocol in an active status beyond the expiration date noted above, you must submit an annual report, checking the box to request a deadline extension for an additional year.

If you have questions about these procedures, or need any additional assistance from the IRB, please contact the IRB point of contact, Ms. Jasmine Trana (305-899-3020 or jtrana@barry.edu)**.** Finally, if you are required to carry professional liability insurance, please review your policy to make sure your coverage includes the activities in this study.

Sincerely,

Tan Fung Ivan Chan, EdD, OTD, OTR/L
Co-Chair, Institutional Review Board
Barry University
College of Nursing and Health Sciences

****

Note: The investigator will be solely responsible and strictly accountable for any deviation from or failure to follow the research protocol as approved. Barry University has no liability related to claims arising from said deviation or failure.